THE ELEMENTS OF ARITHMETICKE

most methodically deliuered.

Written in Latine by C. VRSTITIVS professor of the Mathematickes in the Vniuersitie of Basill.

AND

Translated by THOMAS HOOD, Doctor in Physicke, and well-willer of them which delight in the Mathematicall sciences.

Obedire nouit, nescit seruire Virtus.

LONDON
Printed by Richard Field.
1596

ELEMENTS
OF ARITHMETICKE

LONDON
Printed by Richard Field.
1596

TO THE RIGHT WORSHIPFVL Sr CONIER CLIFFORD KNIGHT,

health and felicitie.

VLYSSES (Right worshipfull) hauing spent some time in the Court of Æolus, where he was friendly entertained, desired at length to be dismissed: his sute was granted, and he sent away in Princely manner, with all the windes tied vp in a bottle, to be commaunded as he thought best. He was not past a dayes iourney or two from the shore, but he went to wracke thorough the misdealing of those, to

whoſe ſafe conduct he had committed himſelfe, and by the winds was driuen backe againe to the coaſt of Æolia. His weather beaten men, and himſelfe being refreſhed as time & place then ſerued, he ſet forward to Æolus his Court, hoping to find ſuch entertainment as he found before. Not daring ſodainly to enter in, he reſted himſelfe at the Porter his lodge, where he heard theſe words:

πῶς ἦλθες Ὀδυσσεῦ, τίς τοι κακὸς ἔχρασε δαίμων;
ἦ μέν σ' ἐνδυκέως ἀπεπέμπομεν.

It may be they moued his patience a little, becauſe they touched his former eſtate, yet hauing better conceite of others in that place he let them paſſe, and preaſed forward to the preſence of the King, ſitting not farre off with his Nobilitie about him, to whom he ſpake

thus

thus in humble manner:

ἀκέσασθε φιλοί. And that his requeſt might the better preuaile, he added his reaſon, (For it is not likely though his eſtate were bad, that he would craue any thing more then reaſon.) δύναμις γὰρ ἐν ὑμῖν. But he was ſo farre off from finding his expected reliefe, that he was commaunded to get him thence with theſe hot wordes to his cold comfort:

ἔρρ' ἐκ νήσου θᾶσσον ἐλέγχιστε ζωόντων.
οὐ γὰρ μοι θέμις ἐστὶ κομίζεμεν, οὐδ' ἀποπέμπειν
ἄνδρα τὸν ὅς κε θεοῖσιν ἀπέχθηται μακάρεσσιν.
ἔρρ', ἐπεὶ ἄρα θεοῖσιν ἀπεχθόμενος τόδ' ἱκάνεις.

Such as Vlyſſes his departure was from the Iland Æolia, ſuch was mine from London Northward; ſuch as his entertainement was at his returne, ſuch was mine cõming thither againe. For miſcarying in that voyage beſides mine expecta-

tion (I would say my desert were it not presumption) I found that of Horace to be true:

——— *Diffugiunt cadis,*
Cum face siccatis amici,
Ferre iugum pariter dolosi.

Some said they could not, some durst not for feare, and some answered they would not helpe to repaire that losse, which the vnkind Northerne blast had inforced me to. VVhat should I do, or what other thing could I doe in this case, but that which Vlysses had donne before? ἔτλων καὶ ἐμεινα. I suffered all, and bare it out, not lying still as a man careless, but knowing mine head and handes (vnder God) to be my best friends, I set them to their old occupation againe, teaching the Mathematicall arts, and penning, or translating such books as I thought most conuenient for

that

that purpoſe, of the which this booke is ſome little part. And for ſo much (Right worſhipfull) as I intended therein not onely your pleaſure, but your profit eſpecially among the reſt, I am bold to make that intent knowne in dedicating it vnto your VVorſhip, & crauing your fauorable acceptance therof, which if it pleaſe you to afford, you ſhall inuite me to a farther worke, and bind me not to be vnthankfull for your friendly fauour.

Your worships to commaund,

THOMAS HOOD.

DEDICATORIE.

that purpose, of the which this booke is some little part. And for so much (Right worshipfull) as I intended therein not onely your pleasure, but your profit especially among the rest, I am bold to make that intent knowne in dedicating it vnto your VVorship, & crauing your fauorable acceptance therof, which if it please you to afford, you shall inuite me to a farther worke, and bind me not to be vnthankfull for your friendly fauour.

Your worships to commaund,

THOMAS HOOD.

THE ELEMENTS OF ARITHMETICKE MOST METHODICALLIE DELIVERED.

CHAP. 1.

The definition and subiect of Arithmeticke.

ARithmetick is the Art of numbring well.

This definition is short & plaine, being drawen from the end of Arithmetike, after which sort other Arts also are defined.

To number vvell, is to expresse and practise the vvhole force, properties, and vse of numbers.

It appeareth then that the Subiect of Arithmetike (that is to say, the thing whereabout that Art is wholly occupied and whereunto all the rules thereof are to be referred) is number.

Number is a Multitude consisting of vnities.

It is the definition of the seuenth booke of Euclide. Aristotle in his Categories seemeth

to define it thus. A number is a diſcreet quantitie not hauing poſition but order. The which definition agreeth with that of Euclid, ſauing that it diſtinguiſheth an vnitie from a point by a certaine contrary qualitie which is in them; attributing to a point poſition or place, to an vnitie only order & conſequence. Whereby it appeareth that a number is the ioyning togither of things following one an other in order not touching one the other much leſſe cleauing togither, and that in ſuch ſort that you can not determine any bond or common knot in them, whereby they ſhould be knit togither and continued.

The beginning of number in an Vnitie.

An vnitie is that, whereby euery thing that is, is ſaid to be one.

An vnitie properly is no number, becauſe it is no multitude, for multitudes onely are numbred: neither is it a part of a number becauſe that euery part of a number ought alſo to be a number. It anſweareth in proportion to a moment, and to a point: whereof the one is the beginning of time, the other of magnitude, and yet no part of them: as Ariſtotle proueth in the fifth and in the beginning of the ſixth booke of Phiſicks: yet in Arithmeticall numeration it is wont to be taken for a number, and that the leaſt that may be.

The diuiſion of number ſhall be drawen out of numeration.

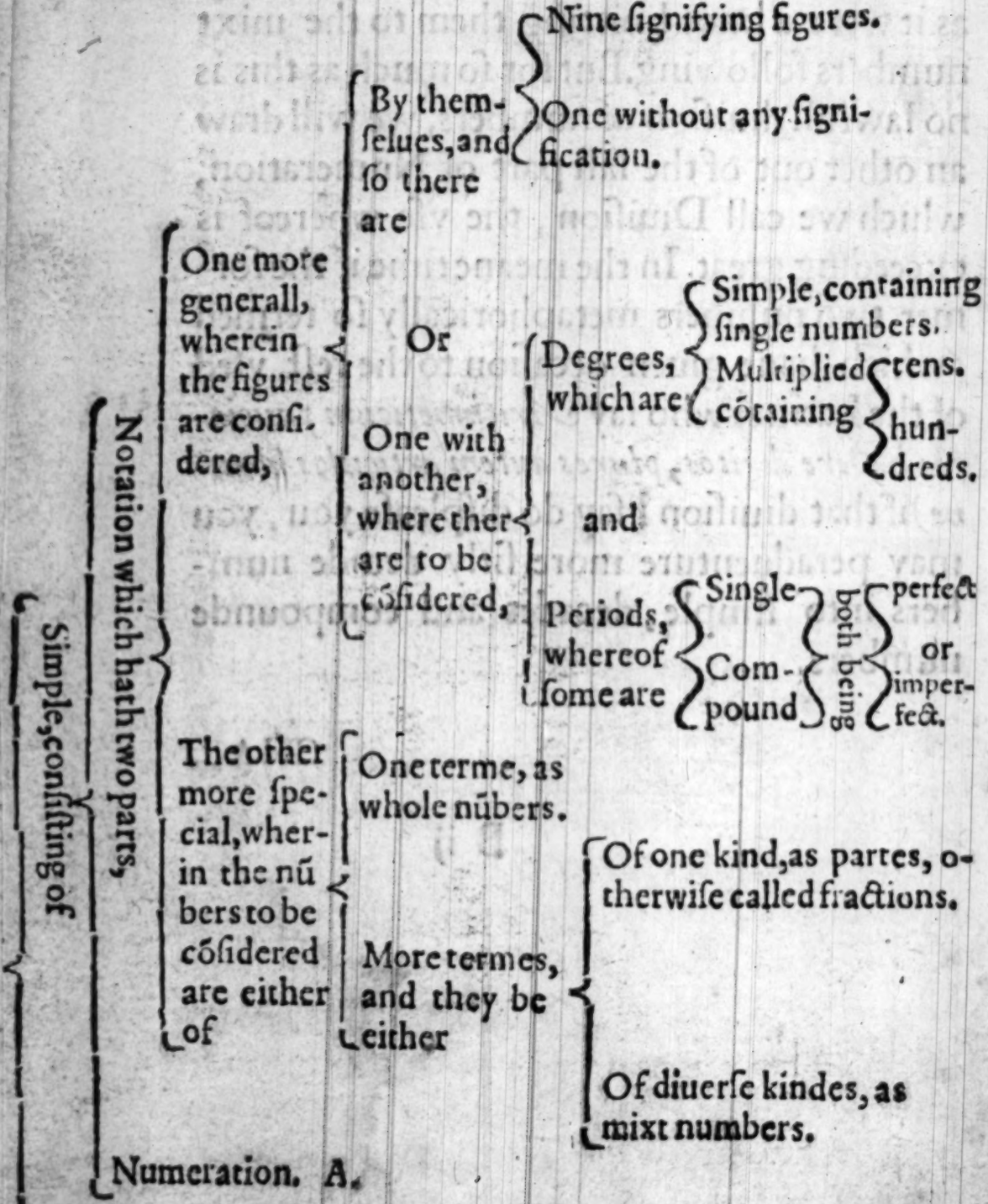
The Scholemen deuide numbers into Digites

gites, articles & mixt numbers: to this end as I think that they might make a distinction in the naturall consequence of numbers: wherin first of all the nine simple figures are to be cõsidered, the which the number 10. succeedeth as it were a knot knitting them to the mixt numbers following. But for so much as this is no lawfull diuision of numbers, we will draw an other out of the last part of Numeration, which we call Diuision, the vse whereof is exceeding great. In the meane time if the former two numbers metaphorically so termed (which haue giuen occasion to the rest vsed of the Latines who say *Arithmeticum pauciores habere digitos, plures autem articulos homine*) if that diuision I say do displease you, you may peraduenture more fitly deuide numbers into simple, decades and compounde numbers.

CHAP.

CHAP. 11.

Of Notation, and of the first part thereof.



ARithmeticke is double: Simple and compared, Simple is that vvhich teacheth the practiſe of ſimple numbers vvithout compariſon.

This partition ſpringeth of the diuerſitie of the ſubiect. For numbers are conſidered either by themſelues alone abſolutly, or elſe one with an other in reſpect of their proportionalitie, or proportion which they haue one to an other. Hereon commeth this foreſayd diuiſion containing all things belonging to this Art.

Simple Arithmeticke conſiſteth of two parts, Notation and Numeration.

Notation is the vvay hovv to ſet downe numbers in vvriting, and to expreſſe them being vvritten.

The Greekes called it Semeioſis, and it is more fitly called Notation then Numeration, becauſe it teacheth vs how to note, that is, to write euery number, and to expreſſe it when it is written.

There be two parts of it. The firſt expreſſeth generally the value of the figures vvherewith the numbers are vvritten.

The figures vvherewith numbers are written are ten.

As 1.2.3.4.5.6.7.8.9.0. Neither the Greekes nor the Latins vſed theſe figures. For the Romanes vſed onely theſe ſeuen letters, M.D.C. L.X.V.I but the Grecians vſed all the letters of the Alphabet, being marked with certaine

prickes. Many therefore thinke that theſe figures were inuented by the Arabians, the which people in old time practiſed this Art and the whole ſcience of the Mathematickes very diligently, whereof beſide the word Algorithme, whereby they expreſſed the Art of numbring, many other words left vnto vs in this kinde of knowledge may be a ſufficient teſtimonie. For the Latins in the former rude and barbarous age forſaking the wel-ſprings of the Grecians, followed after the ſmall riuers of the Arabians. Valla and Cardane thinke them to be inuented by the Indians. Some there be that attribute thē to the Phœnicians. They haue bin retained by the common conſent almoſt of all people, becauſe they be moſt conuenient to number withall, and moſt fit to receaue all kinde of practiſe vſed in this Art.

The figures are conſidered eyther by themſelues alone, or one with another. By themſelues alone, when they are without compoſition, or as it were without compoſition, for then the firſt nine haue a certaine value.

The tenth which hath the forme of a circle ſignifieth nothing ſtanding by it ſelfe alone, but being placed with the reſt, it maketh to the increaſe of their value.

As, 1. is aſmuch as one, 2. two, 3. three, 4. four, 5. fiue, 6. ſix, 7. ſeuē, 8. eight, 9. nine, 0. of it own nature is without value, yet hauing his place wherein none of the other ſignifying figures

is placed, it encreaſeth the value of thē which follow: for it is neuer ſet in the laſt place. It is called commonly a Cyphar.

Figures are conſidered one with an other being ioyned together, and then the proceſſe and order of their places encreaſeth their value.

For ſo much as euery number may be encreaſed infinitely, therefore by reaſon of their diuers encreaſe amoūting to tens, hundreds, thouſands ten thouſands, &c. it is vnpoſſible to write euery one of thē with a diuers figure. For then ſhould the varietie of the figures be alſo infinite. But what is wanting in the figures, that their places through often repeating of them, maketh vp. In continuance of which places the figures take vnto them a diuers ſignification, & that ſo much the greater by how much the more in ioyning the numbers togither they come nearer to the left hād.

The places of numbers are diſtinguiſhed by degrees and periods. A degree is a place whereby the value of each number is gathered.

A degree is of two ſorts: the firſt is of ſimple numbers, the other of multiplyed numbers, to wit, the ſecond degree is of tens, the thirde of hundreds, and ſo foorth after the ſame manner in the reſt.

Numbers according to the cuſtome of all people proceede from the left hand toward the right, ſo that the leaſt number occupieth the laſt place next to the right hand. Wherefore the laſt figure in value is

the firſt, & euery figure in the firſt place keepeth his firſt and proper ſignification. In the other places euery one is armed with a double power, to wit, with his owne and with an other borrowed for a time. For this conſideration is generally to be obſerued in the places: that, that which followeth is twiſe as much as that which went before.

As for example: the figure 4. by it ſelfe alone is foure: and ſo much alſo it ſignifieth in the firſt degree. But being remoued into the ſecond, it ſignifieth fortie, as you ſee here, 40. Againe, being ſet in the third degree it is ten times ſo much as it was in the ſecond: that is, foure hundred, as here, 400. And ſo forth after the ſame manner both in the other figures and degrees.

A periode is that vvhich comprehendeth the degrees, and is eyther perfect, conſiſting of three degrees, or imperfect contayning leſſe then three.

To conclude, that Period vvhich conſiſteth but of one trinitie or ternarie of degrees is ſingle, but that vvhich conſiſteth of more then one, is compound.

As 1 2 3 is a perfect ſingle period: 1 2 3 4 5 6 is a perfect compounde periode. Alſo 1 2 is an vnperfect ſingle periode: 1 2 3 4, 1 2 3 4 5 is an vnperfect cõpound period, 1 2 3 4 5 6 7 8. is an vnperfect triple periode. In numbers the degrees are to the periodes, as the commata & cola are the periodes in an oration.

CHAP.

CHAP. III.

Of the other part of Notation.

THus much concerning Notation in generall: now followeth the other part which expresseth the numbers vvritten more particularly.

And here vve haue to deale vvith numbers, either of one terme, or more. Whole numbers are vvritten vvith one terme.

As 20, 24, 360, 1234, 1578, they be also called numbers of one bande or rancke.

Hovv vvhole numbers of a single periode should be set downe, may be gathered by that vvhich hath beene saide before. But that the compound numbers may be expressed, they must vvith prickes or small lines be deuided into single periods, and be so vttered that at the first pricke vve name a thousand, at the second a thousand thousand, at the third a thousand thousand thousand, and so forth going forward after the same order, till vve haue numbred the vvhole.

Numbers be they neuer so great may easily be expressed, if distinguishing them with periodes you vtter them by partes, in such sort that both the value of the figures and the power of the degrees, may gouerne them when they be vttered. So that at the end of the first periode you shall name a thousand, at the end of the second a thousand thousand (which some call a million) and to be short, at the end of the third periode, you shall name

a thouſand thouſand thouſand: obſeruing the ſame encreaſe in all the periodes following, be they neuer ſo many. Hereupon it commeth to paſſe that the leaſt ſignifying figure of the periode following, is greater then any of the periode going before. Moreouer the ready vttering of numbers maketh a man to write them ſpeedily, & contrariwiſe the ſpeedy writing of them maketh a man to vtter them readily. For example, hauing diſtinguiſhed this number 34·567·890 with prickes you ſhall expreſſe it thus beginning at the left hand. At the firſt periode you ſhall ſay thirtie foure thouſand thouſand or elſe thirty foure millions. At the ſecond, fiue hūdred ſixtie ſeuen thouſand. At the third eight hundred and nintie.

The number following you ſhall expreſſe thus 2·016·542·009·873. Two thouſand thouſand thouſand thouſand: then ſixteen thouſand thouſand thouſand, fiue hundred fourtie two thouſand thouſand, 9. thouſand eight hundred ſeuentie three.

The Romaines were wont to number by hundred thouſands. The which faſhion if you liſt to follow you ſhal ſet down a pricke next to the ſecond degree of the ſecond periode & expreſſe the other numbers following, as you did the firſt. As in this number 77·89·320, you ſhall ſay thus ſeuenty ſeuē hundred foure ſcore and nine thouſand three hundred and twentie. After this manner Plinie in his ſecond

cond booke 1 0 8. Chap. affirmeth according to the saying of Artemidorus that the longitude of the earth inhabited Westward from the pillers of Hercules in India, is eighty fiue hundred seuentie and eight thousand miles. The which number must be set downe thus.

85|78|000.

Thus much for the notation of numbers of one terme: now follow the numbers of manie termes. And they be eyther of one kinde as partes (which we commonly call fractions,) or of diuerse kinds as mixt numbers.

Partes are set downe with two termes deuided one from an other with a line: whereof the vppermost numbreth the parts, the neathermost nameth them.

Although an vnitie by it self of it own nature can not be deuided, yet if we regarde the subiect thereof, or the magnitude whereto it is applied, it may be deuided. And yet the vnities of the number proceeding of this diuision do not arise of the very vnitie it selfe, but of the deuiding of the magnitude, that is: they come not of the vnitie as it is by it selfe alone, but as it is ioyned to some other thing. As a shilling can not be deuided as it is one, but as it is a peece of money, which customablie is deuided into testons, groates, three pences, two pences, pence and half pence, &c. Wherfore although partes be called by the name of numbers, and are written with the figures be-

longing to whole numbers: yet in very deede they are no numbers, if Euclides definition of a number be true. But in the notation of theſe partes there are two things to be conſidered, the quantitie of the parts (for a whole number may haue many parts, both great and ſmall) and the number of them. Hereupon it commeth that when they be written they require two termes or numbers: whereof the vppermoſt declareth the number of the parts the which are to be found out, whereupon it is called the Number, or Numerator. But the neathermoſt naming the parts of the whole, expreſſeth their quantitie: wherefore it is called the name, or the Denominator. As $\frac{1}{2}$ ſignifieth one part of the whole deuided into two halfes, that is; one halfe. $\frac{1}{3}$ one third part $\frac{3}{4}$ three fourth partes, or three quarters. $\frac{5}{6}$ fiue ſixth partes, that is, fiue partes of the whole deuided into ſixe parts.

Hereby it appeareth that the greater the Denominator is, the leſſe is the quantitie of the partes.

Partes are eyther principall, or of the ſecond ſort. The one haue their originall of the firſt diuiſion of the vvhole, the other are parcels of the principall parts.

They may be called Simple & Deriuatiue parts: whereof the one ariſeth of the firſt diuiſion of whole numbers, the other of breaking the parts into ſmaller parcels. Their notation and value differeth much. The princi-

pall

pall parts are written after the maner before named. But the parcels, or the parts of parts called commonly fractions of fractions, are set downe by the first toward the left hand, without any line betweene them for difference sake, as $\frac{5}{12}\ \frac{1}{3}$ fiue twelue parts of one third part, $\frac{3}{4}\ \frac{7}{8}$ three fourth parts, of seuen eight parts: or else otherwise by the prepositition $\frac{5}{6}$ out of $\frac{3}{1}$.

Moreouer, partes are eyther proper or improper. They are lesse then the vvhole: these are eyther equall vnto, or greater then the vvhole.

True and proper partes in deede are lesse then the whole, and therefore that which they signifie can not be expressed by an vnitie, whereupon they haue the numerator alwaies lesse then the Denominator. But those parts whose Numerator is equall or bigger then the Denominator, are improper partes, for that they may be expressed by an whole or mixt number: as $\frac{3}{3}$ are as much as one whole: $\frac{4}{2}$ are two whole, $\frac{6}{5}$ are one whole, and $\frac{1}{5}$ more then the whole.

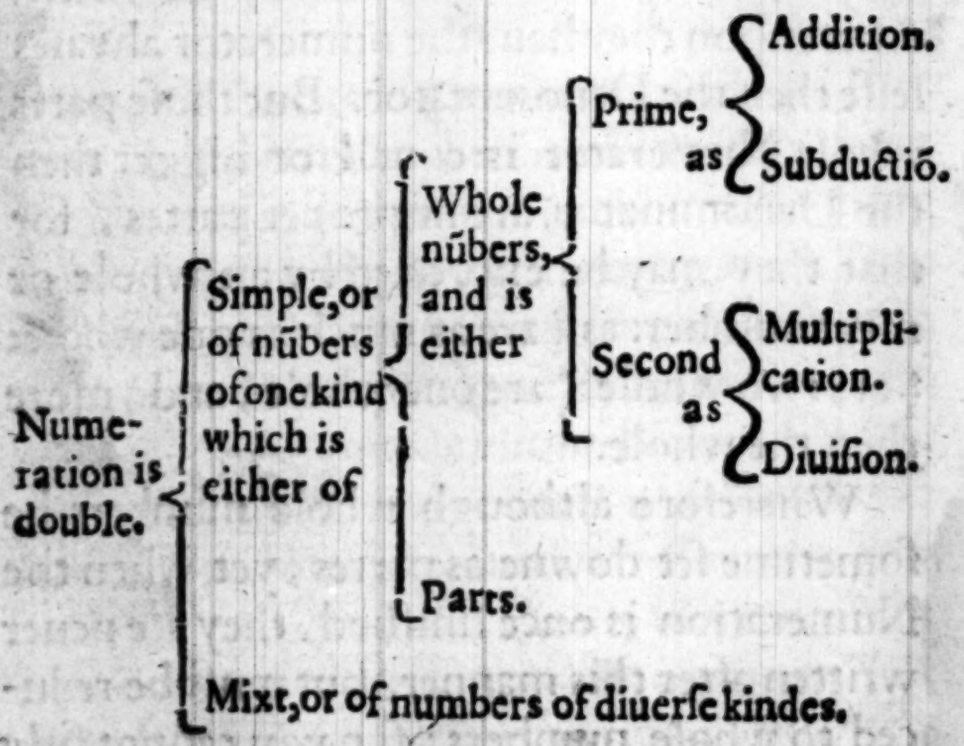
Wherefore although whole numbers be sometime set downe as partes, yet when the Numeration is once finished, they are neuer written after this manner, but must be reduced to whole numbers: for you cannot fitly say, $\frac{2}{2}$ two halfes, or $\frac{4}{3}$ foure third partes, or $\frac{8}{4}$ eight quarters: but one whole, one whole with a third part, &c.

There remaine as yet mixt numbers, which are whole numbers with partes. But their notation may be easily gathered out of the former kindes.

In Geometrie they are not accounted for numbers: but when the measure of a thing cannot be expressed but by a mixt number, it is called irrationall, as though it were not indeede to be expressed by a number. Herevpon commeth the name of Surde numbers.

CHAP. IIII.

Of Numeration, the other part of simple Arithmeticke, and of the first kind thereof.



Hitherto we haue entreated of Notation, nowe followeth Numeration: which of two

two numbers giuen findeth out the third.

As the framing of an argument is in Logicke, euen ſo is numeration in this art. For as the Logicians in reaſoning doe inferre the concluſion by the premiſſes, euen ſo the Arithmeticians by numbering doe inferre a diuerſe number from the numbers giuen.

The numbers giuen are either of one and the ſelfe ſame kind, as whole numbers, or partes, or of diuerſe kinds as mixt numbers.

Numeration of whole numbers is when the numbers giuen are whole numbers.

And this Numeration is double. Prime: which numbreth one number with an other onely once, and that eyther by adding or ſubtracting them: whereupon it is called Addition or Subtraction.

As if theſe numbers 360. and 15. were giuen either to be added or ſubtracted, you muſt either adde 360. once to 15. or elſe ſubtract them once onely, and not many times as in the ſecond numeration. Wherefore this may well be called ſingle numeration, and the other manifold numeration.

Addition is a Prime Numeration, which ioyning one number to an other, findeth out the totall of the numbers giuen.

Touching Addition of ſimple figures the rule of reaſon teacheth vs how to doe it, but the addition of compound figures may be learned by this rule.

If, proceeding orderly from the firſt of the

numbers giuen to thoſe which folow, you ſet the particular ſummes (being leſſe then ten) of the figures of euery degree added together alone by themſelues, each one vnder his owne degree, putting the ouerplus (that is, for euerie ten, an vnitie) to the degree following: you ſhall finde out the totall of the numbers giuen.

Addition of ſingle nũbers may be wrought by the direction of nature only, wherefore it may be practiſed in our mindes, and brought hither for the ſpeedie diſpatching of our worke. As 5 and 6 are 11, 7 and 8 are 15, 9 and 9 are 18. But the Addition of compound numbers goeth on by peecemeale frõ part to part, ſo that it requireth the helpe of art. The firſt precept to be obſerued in this place, is to number all the figures of euerie place alone by themſelues, as though they were but ſingle figures: and this muſt be done in all the partes of Numeration following. The ſeconde is to number the figures of one and the ſame degree onely together, that you confound not the one with the other. Laſt of all, you muſt ſet downe the totall made of the parts, in ſuch ſort, that if they increaſe to ten or more, onely the figure next to the right hand muſt be written vnder that degree, the other which remaineth muſt be added to the degree following. As for example, adde 983 to 402. Firſt, ſetting downe the numbers in ſuch ſort one vnder an other, that the figures

of

of euery degree may answere one to an other, I must beginne at the first and say, 2 and 3 are 5, the which I set downe vnder that degree: then 0 and 8 are 8, the which I write vnder the second place. Then 4 and 9 are 13, which number for that it is in the last place, I set it downe whole. So that the totall of the numbers giuen is, 1385. The example standeth thus.

```
  9 8 3
  4 0 2
-------
1 3 8 5
```

Item adde 89647 vnto 78450. The numbers being duely set downe one vnder an other, according to the order of their degrees, and a line drawne vnderneath them, I begin to number at the right hand saying, 0 & 7 are 7. Then 5 and 4 are 9. Item 4 and 6 are 10. But for so much as the totall is written with two figures, I set downe the cypher only vnder that degree, and keepe the vnitie in my minde. Then 8 and 9 are 17, and the vnitie which I kept maketh 18, where againe I set downe 8 because it is next to the right hande and keepe the vnitie next to the left hand in my minde. Lastly 7 and 8 are 15, and one which I kept are 16, the which I set down wholly. So the totall of the numbers giuen is, 168097. The example is to be set downe in this order.

89647
.78450
168097

In this part of numeration and in the reſt, there are ſaid to be but only two numbers giuen, although ſometime there be more to be added together, for that the two firſt numbers onely are added together, and then if there be more, the totall is added to that which followeth, as in the example following.

90641
4790
2853
98284

678921
54086
560
9522
743089

Here, I ſay, 2 and 6 are 8, 8 and 1 are 9, taking 8, which is the totall made of 2 and 6, for one ſimple number, and adding it to 1 ſtanding aboue it, &c.

Subduction is a prime Numeration, vvhich taking one number from another, findeth out the Remainder of the numbers giuen.

The vſe of it is to find out the difference betwene two numbers: whereupon it is requiſite that the numbers giuen, ſhould be vnequall, and that from which the ſubtraction is made, muſt be the greater.

Subduction of ſimple numbers may be drawen out of the table of our mind. But cõpound nũbers are to be ſubducted by the Theoreme following.

If,

If, proceeding orderly from the first of the numbers giuen, you set the remainder of euery figure of one & the same degree, subtracted one from the other (either by it selfe alone, or by borrowing ten of the degree folowing) seuerally ech one vnder his own degree: the remainder of the numbers giuen may be found.

In euery part of numeration there must a certaine practise go before it: the which must not then be to seeke when we should worke, but it must be drawen out of the threasure of our minds, and ministred vnto vs. So that in subductiõ there is required of vs a ready foresight, that we may know what nũber will remaine, when any one of the figures is taken frõ another by it selfe alone, or els added vnto ten. As 5 from 8, there remaine 3, 4 from 9 there remaine 5. Or by adding ten to the single figure: as 9 from 17, there remaine 8, 6 from 15, there remaine 9, &c.

In compound and great numbers, it is most conuenient to place the greater vppermost, and the lesse beneath, and then to number by the parts going on towarde the left hande. For that way is more easie then the other which proceedeth from the left hande to the right, as in the examples following.

857	58734	300065
345	9832	1984
512	48902	298081

In the firſt example euery thing is plaine.

In the ſecond I beginne to number, 2 from 4 remaine 2, 3 from 3 remaine 0. Then for ſo much as 8 can not be taken from 7, the figure of the ſame degree, I borrow an vnitie of the degree following, and ioyne it to 7 thus 17, and drawe 8 out of 17 there remaines 9 to be noted vnder neath. This may be done, if (as ſome are wont) I take the diſtance of the neather number from 10. & adde it to the vppermoſt number, or ſet downe the vppermoſt number only, if the neathermoſt be 10. As for example, becauſe 8 differeth 2 from 10, I take 2 and adde them to 7 which make 9 to be written vnder the third degree. But for ſo much as I borrowed 1 of 8 in the vppermoſt number following, there remaine but 7, I draw therefore 9 from 17 (taking an vnitie from 5 following) there remaine 8. In the laſt degree, becauſe there is no figure beneath which aunſwereth to that aboue, I ſet not downe the whole 5 but 4, becauſe I borrowed one of it before.

In the third example, that the ſignifying figures may be ſubducted out of the cyphers, we muſt borrow of the numbers following an vnitie, as we did before. Firſt take 4 from 5 there remaines 1, then becauſe you cannot take 8 from 6, you muſt ſubtract it from 16, there remaines 8. For although the figures following, from whence I borrow the vnitie, be a cyphar, and therefore without ſignification

tion

tion, yet for so much as it hath a signifying figure before it, I may borrow an vnitie of it. Then I go on and take 9 from 9, there remaineth nothing. Moreouer I take 1, not from 10, because there was an vnitie borrowed of it before, but from 9, there remaine 8. And for so much as there is no figure to be drawen out of that which followeth, I set downe 9. Likewise I write downe the last figure lesse by one then it is, because I presuppose that it wanteth that vnitie which I tooke from the first cyphar.

CHAP. V.

Of the second kind of Numeration.

HEtherto of prime Numeration. Now followeth second Numeration, vvhich numbreth one number vvith an other, so often as one of the two numbers giuen requireth, and that eyther by multiplying or diuiding: vvhereupon it is called Multiplication or Diuision.

Second Numeration is either often Addition, as Multiplication: or often Subtraction, as Diuision. For one of the numbers giuen is so often eyther encreased or diminished, as the other, eyther the multiplyer or diuisor requireth, according to the number of the vnities contayned in them.

Multiplication is a seconde Numeration, vvhich ioyning together the multiplicande, so often as there bee vnities in the Multiplier.

bringeth foorth the facit.

This is the 15. definition of the 7. Booke of Euclide. It maketh no matter which number you make the multiplicande, or which number you make the multiplier, as it appeareth by the 16. prop. of the ſeuenth Booke of Euclide, which ſayth: If two numbers multiplied together, the one into the other, produce any numbers, the numbers produced are equall the one to the other: that is, they make one and the ſame number. Notwithſtanding the Scholemen ſet the greateſt number vppermoſt for the multiplicande, and the leaſt neathermoſt for the multiplier, for that it ſeemeth moſt conuenient for young beginners.

Multiplication of ſingle numbers is to be conceiued in our mindes by this Theoreme.

If eyther of the two numbers giuen be diuided into certaine partes, the product comming of the whole numbers multiplied together, is equall to the product made by one of the whole numbers, and the parts of the other number ſo diuided. The 1. propoſition of the ſecond booke, or elſe, if both the numbers giuen be diuided into certaine parts, that which is made of the whole number, is equall to the product made of their parts.

That we may multiply eaſily and readily, we muſt haue in our mindes, a table whereby we may know what the pruduct of euery ſimple figure is, being multiplied one by an other: the which thing may be eaſily done by the

the rule of whole numbers and their partes. Euery child by the direction of nature can tell how many twiſe foure, or foure times fiue, or thriſe ſixe doe make. But if you happen to ſticke in greater numbers, a litle exerciſe will make this table very readie. As if you would knowe how much ſeuen times eight is: diuide eyther of the two numbers giuen, into as many partes as you liſt, as 7. into 2. 3. 2. Then multiply 8. by euery one of theſe parts, and adde the particular productes together, and you ſhall haue 56. How many are eight times 9. diuide 9. into 3. 3. 3. and multiply 8. by thoſe partes, adding the products together, ſo you ſhall make 72. The ſame may be done if you diuide both the numbers. The examples muſt be ſet downe thus.

I.

8	
2. 3. 2.	7
16	
24	
16	
56	

8	
3. 3. 3.	9
24	
24	
24	
72	

II.

44	8		53	8
34	7		54	9
16			12	
16			20	
12			15	
12			25	
56			72	

The Scholemen frame this Table By this Theoteme. Two numbers being giuē which ioyntly together are more then tenne, if you multiply the difference of each of them from tenne one by the other, & then ſubduct croſſwayes one of the differences out of one of the numbers giuen, the product and the remainder parted into diuerſe degrees, ſhall be the product of the numbers giuen. As you ſee in theſe examples.

The numbers giuen.	Their difference.	Numbers giuen.	Difference.
8	2	9	1
7	3	8	2
5	6	7	2

Multiplication of compound figures is drawen out of the Theoreme following.

If proceeding from the firſt of the nūbers giuen toward the left hand, you multiply the figures of the multiplier, into euery one of the

the multiplicand, and ioyne together the particular products being lesse then ten, setting them orderly vnder their multiplier, and put the ouerplus to the degree following: the totall made of the partes, is the product of the numbers giuen.

For example sake, multiply, 365 by 3. Here the numeration must proceede from part to part. Hauing therefore set downe the numbers, as you did in the former kindes of numeration, multiply the neathermost figure seuerally into euery one of the vppermost thus: three times 5 are 15, I set downe 5, and keepe 1: thrise 6 are 18, and one which I kept, are 19, I set downe 9 and keepe one to be added to the degree following: thrise 3 are 9, and one are 10, which I set downe wholy. The example standeth thus.

365
3

1095

Now take an example of multiplication to be wrought by the parts of both the numbers giuen. Multiply 1568 by 54: Item multiply 3508476 by 2509. In both these numbers you shall haue so many rowes of numbers, as there be figures in the multiplier. Wherefore we must take diligent heede, that we confound not the particular products, and beware that in distinguishing the products, we set the first figure of euery one of them

vnder his particular multiplier, as you see here.

1568	3508476
54	2509
6272	31576284
7840	0000000
84672	17542380
	7016952
	8802766284

An abridgement of multiplication.

Neither are the shortest wayes to be neglected: this therefore is an especiall abridgement of multiplication.

If one, or both the numbers giuen, haue cyphers in the beginning, thē if multiplying only the signifying figures together, you put the cyphers to the totall, you shall haue the product of the numbers giuen.

The correllary of this abridgement may be this.

If the last figure of the multiplier be an vnitie, and the other cyphers, then setting the cyphers before the multiplicand, you shall haue the product of the numbers giuen. The exsamples.

360	15000	8734
15	500	100
180	75	873400
36	7500000	
5400		

Diuision

Diuiſion is a numeration, vvhich drawing one number from another, as often as may be, findeth out the quotient of the greater.

In diuiſion there are three numbers to be conſidered, the Diuidende, the Diuiſor, & the Quotient, the which muſt be ſo placed, that the Diuidend may ſtande aboue, the Diuiſor beneath, and the Quotient at the ſide, or betwene them both. The vſe of diuiſion is to declare how many times the leſſe is contained in the greater. The Artificers terme diuiſion by a Geometricall phraſe, calling it *Compariſon*, which is the applying of a meaſure giuen to any right line, as here,

the line C D, is compared with the line A B to ſee how many times it may be cõtained in it. Or it is the ioyning of one ſide to an other ſide, giuen to make a rightangled parallelogram. As therfore in multiplication, the multiplying of two ſides togither maketh a right angled figure, if the lines meete perpendicularly, euen ſo in diuiſion, the diuiding of the area or platforme of the right angled figure giuen by the length as it were by the diuiſor, findeth out the breadth which is repreſented by the quotient, as you ſee here.

Multiply the ſide A C, by the ſide C D, the product will be the area or platforme of the right angled figure A B C D. Likewiſe diuide the platforme of the right angled figure E F G H, by the ſide G H which is the length, you ſhall find out the other ſide E G which is the breadth which anſwereth to the quotient in diuiſion. Hereupon it is called the breadth of the compariſon or of the compared figure.

A quotient is a part of the diuidende, hauing the ſame denomination vvith the diuiſor.

This definition is taken out of the 39 prop. of the ſeuenth booke of Euclide which ſayth: *If a number meaſure any number, the number meaſured ſhall haue a part of the ſame Denomination with the number which meaſureth it. And contrariwiſe, as it is in the 40. prop. If it haue a part, the number vvhereof the part taketh his denomination, ſhall meaſure it.* For Euclide taketh meaſuring here for diuiding: as for example, becauſe 48 may be diuided by 6 into 8 the quotient, I ſay that 8 being the quotient, is the 6 part of 48, and taketh his denomination of the Diuiſor. And contrariwiſe, becauſe 8 is the 6 part of 48, therefore 6 diuide 48. Likewiſe if 6 diuide 54, the quotient will be 9, and 9 therefore is the 6 part of 54, and becauſe 9 is the ſixth part of 54, therefore 6 diuide 54, &c.

A part of a number is a leſſe number in reſpect of a greater, when the leſſe meaſureth the

greater.

greater.

This is the first definition of the 5 and the third def. of the seuenth booke. Euclide will not haue euery number that is lesse then a greater number, to be a part of the greater, but onely that which being taken oftentimes by it selfe alone, measureth the greater, that is, eyther maketh it, or taketh it cleane away: As two is the third part of 6, fiue is the halfe part of ten. For that 2 being three times taken, eyther maketh or taketh away 6, and 5 twice takē make 10 or destroyeth ten. This kind of part is called commonly, *pars metiens*, or *mensurans*. A measuring part and of the barbarous it is called *pars aliquota*, an aliquot part, that is, a quotient: but the number which by it selfe alone being oftentimes taken, maketh not a number without the helpe of some other part or number, is the parts of the whole, vsing the worde parts in the plurall number for distinction sake. As 2 are the parts of fiue, because two make not fiue alone, without the helpe of three, or some other numbers. This kind of part they commonly call, *pars constituens*, or *componens*: of the barbarous it is called, *pars aliquanta.*

Diuision of a simple quotient may be taken out of the table of Multiplication. For by the same numbers that a number is made, by the same it may be diuided.

The first principle to be obserued in diuision is to consider wittily, and to know rea-

dily by what number, and what number euery one of the ſingle figures do diuide another, as ſixe times 7 are 42, therefore 7 diuide 42 by 6, and 6 diuide it by 7. Item ſeuen times 9 are 63, therefore 9 diuide 63, and the quotient is 7, or 7 diuide it, and the quotient is 9.

The diuiſion of a number that hath many figures, is wrought by the Theoreme following.

If, beginning at the laſt figure of the numbers giuen, you multiply the particular quotient of the diuiſor contayned in the diuidend (ſetting it downe aſide by it ſelfe) into the Diuiſor, and then ſubtract the product from the diuidend, doing this as often as may be, by ſetting forwarde the Diuiſor towarde the right hand, till you come to the firſt figure of the diuidend, the quotient of the parts will be the quotient of the numbers giuen.

Although diuiſion commonly be wrought by finding out the quotient by multiplication & ſubduction: yet the principall worke wherein the whole force or vertue of diuiſion conſiſteth, is onely the finding out of the quotient, which being once found, the diuiſion in minde is ſuppoſed to be done. But when we haue found out the quotient, we are to thinke and conſider with our ſelues into how many partes the diuiſor cutteth the number which is ſet ouer it, and how often it may be drawen out of it. That this therefore may be ſet before our eyes, and that in remouing for-

forward the Diuiſor, the remainder (wherein afterwarde the quotient muſt be ſought for) may more manifeſtly appeare, the truth of that which we conceiued before, is proued by multiplication and ſubtraction.

To expreſſe this by examples, diuide 4936 by 4, we muſt ſeeke how often the Diuiſor 4 being ſet vnder the laſt figure of the diuidend, is contayned in the ſame. I ſay then that 4 is contayned in 4 once: wherefore I ſet downe 1 in the quotient, and take 4 out of 4, there remaines nothing. Then ſet I the Diuiſor one degree forwarde towarde the right hand, where 4 are contayned in 9 twice: wherfore I note 2 in the quotient, and multiply the Diuiſor by it, the product is 8, which being ſubducted from 9, there remaine 1. The figures out of which the ſubtraction is made, muſt ſtraight way be blotted out. Againe ſet forwarde the diuiſor: there 4 may be taken from 13 three times, wherefore I ſet 3 in the quotient, and multiply 4 by it, the product is 12, which being drawen out of 13, there remaines 1. To conclude, I find 4 in 16 foure times, therefore I ſet downe 4 for the quotient, and multiply 4 by it, which being taken out of 16, there is nothing left. The example.

```
  1 1                          1 1
4 9 3 6                      4 9 3 6
-------------(1234. or thus,  -------
4 4 4 4                      1234.
  8 2 6                      -------
  1 1                        4 4 4 4
                               8 2 6
                               1 1
```

Againe deuide 1008 by 36. In this example there be many things to be taken heede of. First, because the last figure of the diuidend, is lesse then the last of the Diuisor, therfore I set not 2 vnder 1, but one place farther towarde the right hande vnder the 0. And first I consider with my selfe, how often three may be had in ten, I find it to be thrise, and there remaineth one. But for so much as I cannot subduct the Product made by multiplying the quotient into the whole diuisor, out of the number standing aboue it, to wit, 108 out of 100, therefore I take a quotient lesse then that by one, that is 2, whereby the diuisor being multiplied, there arise 72, which being taken out of 100, there remaine 28 to be written ouer the head. Then the diuisor being set forward, I see three to be contained in 28 nine times, but because I must haue regard also of the figure following, I set but 8 in the quotient, which being multiplied into the diuisor, make 288 to be drawen out of the number set ouer it. The example is thus.

```
  28                    28
 1008                  1008
 ----                  ----
  366 ( 28 or thus.      28
  728                  ----
   3                    366
  28                    728
                         3
                        28
```

It appeareth therefore by this, that the question must be made not of the whole Diuisor, vnlesse it be a single number, but only of the last figure of the Diuisor, and euery particular must be but a single figure, as if you shold seeke how often 2 were in 21, you can not take it tenne times, but 9 times at the most. Moreouer the nature of the thing requireth that we find out alwayes such a quotient, as being multiplied by the Diuisor, maketh no greater a number then the diuidend is. See the examples following.

```
 299552          1812784
 ------ (407     ------- (1096
  736             1654
```

An abridgement of diuision.

If the Diuisor end in cyphers, the worke may be wrought by the signifying figures alone, setting the cyphers in the meane time, vnder the vtmost figures of the diuidend, next to the right hand. But if the last figure of the Diuisor be an vnitie, and the rest cyphers, then setting the Diuisor as before, and

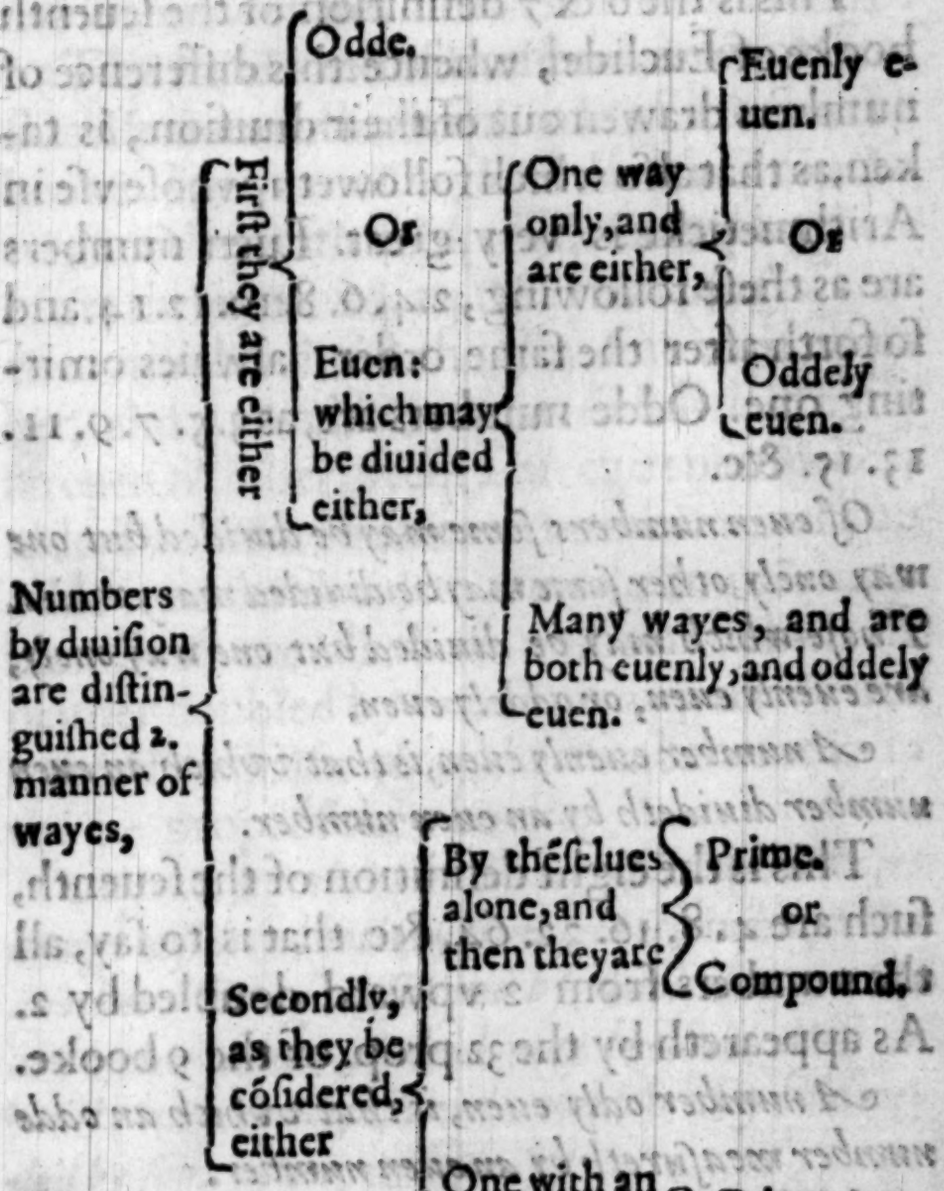
taking the figures that haue no cyphers vnderneath them for the quotient, the diuision is dispatched. The figures that remaine after the diuision is done, must be set downe as the partes are with the diuisor vnderneath them for their denominator.

As for example, diuide 165968 by 360. Item 6734 by 100: the example shall stand thus.

165968 (461 8/360 | 6734 (67 34/100
360 | 100

CHAP. VI.

Of the double diuision of numbers.



NVmbers are diuided two manner of wayes, first into euen and odde numbers. An euen number is that which may be diuided by 2. An odde number is that vvhich can not be diuided by 2.

This is the 6 & 7 definition of the ſeuenth booke of Euclide, whence this difference of numbers drawen out of their diuiſion, is taken, as that alſo which followeth, whoſe vſe in Arithmeticke is very great. Euen numbers are as theſe following, 2.4.6.8.10.12.14. and ſo forth after the ſame order, alwaies omitting one. Odde numbers are, as 3.5.7.9.11.13.15. &c.

Of euen numbers ſome may be diuided but one way onely, other ſome may be diuided many waies. Thoſe which may be diuided but one way onely, are euenly euen, or oddely euen.

A number euenly euen, is that vvhich an euen number diuideth by an euen number.

This is the eight definition of the ſeuenth, ſuch are 4.8.16.32.64. &c. that is to ſay, all the numbers from 2 vpward doubled by 2. As appeareth by the 32 prop. of the 9 booke.

A number odly euen, is that vvhich an odde number meaſureth by an euen number.

As it is in the 9 definition of the ſeuenth booke, Euclide calleth it an euenly odde nūber, as 6.10.14.18.22. For 3 an odde number diuideth 6 by 2 an euen nūber: 5 an odde nūder diuideth 10 by 2 an euen nūber. Such are all the numbers whoſe moytie or halfe is an odde

odde number, as appeareth by the 33 prop. of the seuenth booke.

The numbers vvhich may be diuided manie vvayes, are euenly euen and odde: which may be diuided both by an euen number, and by an odde, into an euen number.

This third kind of numbers, is let passe of Euclide among the definitions of the seuenth booke, but yet not neglected in the propositions of the ninth booke. The examples therof are these, 12. 20. 24. 28. 56. 144. As 2 an euen number diuideth 12 by 6 an euen number, and 3 an odde number diuideth it by 4 an euen number. Item 2 an euen number diuideth 20 into 10 an euen number: and 5 an odde number diuideth it into 4 an euē number. Such are all those numbers which are neither doubled by two from 2 vpward, nor haue their halfe an odde number as it is in the 34 prop. of the 9 booke.

Againe, numbers be distinguished otherwaies being considered both by themselues, and one with another. Being considered by themselues, they are eyther prime, vvhich may be diuided by an vnitie onely: or compound, vvhich may be diuided by some other number.

This is the 12 and 13 definition of the seuenth booke, where Euclide diuideth numbers into prime & compound numbers. The matter of this diuision is taken out of the 34 prop. of the seuenth booke, which sayth: *That euery number is eyther a prime number, or else*

diuided by a prime number: that is, a compound number, as appeareth by the 33 prop. of the same booke. A prime number is that which no other number diuideth besides an vnitie, sauing that it measureth it selfe. It may be called an vncompound number for that it is made of no number: as 2. 3. 5. 7. 11. 13. &c. A compound number is that which some other number maketh, being taken certaine times, as 4. 6. 10. 12. Of this sort are first of all, all euen numbers, then all those odde numbers which in the eleuenth definition of the seuenth booke are called oddely odde: which may be diuided by an odde number into an odde number, as 9. 15. 21. 25. 27. &c.

Numbers compared one with an other, some are Prime in respect one of an other, vvhich can not commonly be diuided by any other number, but by an vnitie.

Other some are compound in respect one of an other, vvhich may be commonly diuided by one or moe numbers.

This difference of numbers compared togeather, is drawen out of the former, and is set downe in the twelfth and eleuenth definition of the seuenth booke. Numbers prime in respect one of an other, haue no common diuisor beside an vnitie, which measureth all numbers: As 3 and 8, 6 and 7, 5 and 12. Therefore when the numbers giuen are prime one to an other, there can no lesse

numbers

numbers be giuen in the ſame proportions. So that prime numbers are the leaſt, and the leaſt are prime numbers, as appeareth in the twentie and twentie foure propoſition of the ſeuenth booke. And therefore, when the numbers giuen are prime, they are alſo the termes of the proportion giuen, as we ſhall ſee afterward. Compound numbers in reſpect one of an other, may commonly be diuided by one, or many numbers, as 3 and 9, 9 and 15: for 3 is the common Diuiſor of them both. Item 12, 18, 24, may be commonly diuided not onely by 6, but alſo by 3 and 2.

If you vvould knowe vvhether the numbers giuen, be prime or compounde one to an other, you may doe it by the Theoreme following.

Two vnequall numbers being giuen, if in ſubducting the leſſe from the greater as often as may be, the remainder diuideth not that which went before it, vntill it come to an vnitie, the numbers giuen are not prime one to an other. 1. prop. 7.

As in 4 and 11, 18 and 7, 35 and 12, in ſubducting the one from the other continually, you ſhall come to an vnitie: wherefore I ſay, that they be prime in reſpect one of an other.

11	18	35
4	7	12
7	11	23
4	7	12
3	4	11
1	3	1
	1	

And hereby we may easily conclude, if after the continuall subducting of the one from the other, we come not downe to an vnitie, but meete with some one number diuiding that which went before: the numbers giuen are compound in respect one of an other. As in 14 and 6, 30 and 18, 49 and 14.

14	30	49
6	18	14
8	12	35
6	(2	14
(3	6	21
2		14
		(2
		7

An abridgement of the former worke.

For so much as diuision is nothing but an often subduction, if you diuide the greater number giuen by the lesse, you may doe the former worke more readily. Yet in diuiding the

the numbers after this maner, you must haue no regard to the quotient, but compare the Diuisor with the remainder. As I proue that 234 and 17 are prime numbers one to the other, by diuision thus.

1	4	1
63	17 (1	13 (3
234 (13	13	4
177		12
11		
5		

In compound numbers take these for example 144 and 27.

9	
144 (5	27 (3
27	9
135	27

CHAP. VII.

Of the greatest common Diuisor, and the least common diuidend.

NVmbers are diuided after the manner before named. Moreouer out of the difference of numbers compared together, there ariseth a double inuention: one is the finding out of the greatest common diuisor, the other of the least common diuidend.

For so much as compound numbers may be diuided often times by many diuisors, the drift of the first inuention is out of many to choose the greatest number, which may be

the common diuiſor, or as Euclide termeth it the common meaſure of them all. The numbers therefore giuen

- Are either
  - Two onely, and no more, as in the firſt Rule. I.
  - Three or more, & in them we meete with the greateſt common diuiſor, either,
    - At the firſt. II.
    - Or at the ſecond time. III.

The finding out of the greateſt common diuiſor is to be learned by the Theoreme following.

I. If after the continuall ſubduction of two numbers giuen, the one from the other, as often as may be (which may more briefly be done by diuiſion) there remaine ſome one number which will exactly diuide them both: The number remaining ſhall be their greateſt common diuiſor.

As in 76 and 20, their greateſt common diuiſor is 4, as appeareth by this.

16	4	
76(3	20(1	16(4
20	16	4
56		

Item

Item of theſe compound numbers 63 and 14, their greateſt common diuiſor is 7.

7
63(4 14(2
14 7
56 14

II. When there are three numbers giuen, if the greateſt cõmon diuiſor of the two formoſt, being compared with the third, do diuide it alſo, that diuiſor ſhall be the greateſt common diuiſor of them all: and ſo forth in the reſt, be they neuer ſo many. 3 prop. 7.

As 14. 21. 35. 63. Compound numbers.
7 The greateſt cõmõ diuiſor.
2. 3. 5. 9. The quotients of the numbers, being prime numbers one to an other.

Here 7 being the greateſt common diuiſor of the two formoſt 14 and 21, compared with the third number 35, diuideth it alſo and in like manner the fourth 63: wherefore it is the greateſt common diuiſor of them all.

III. But when the greateſt common diuiſor of the formoſt, diuideth not thoſe which follow, then comparing the firſt diuiſor, found out with the third number, the greateſt common diuiſor of the numbers compared together ſhall be the common diuiſor of the numbers giuen: and ſo foorth in the reſt be they neuer ſo many. 3 prop. 7.

As betwene 18, 12, 9. The greateſt common diuiſor betwene 18 & 12, is 6, the which nũber

for ſo much as it cannot diuide 9, compare 9 and 6 together by themſelues, and then be-cauſe the greateſt common diuiſor of 6 and 9 is 3, it ſhall be alſo the greateſt common Di-uiſor of all the numbers giuen. The example is thus ſet downe.

The numbers giuen.

The nũbers cõpared.

The greateſt Diuiſor.

See alſo this example following of foure compound numbers in reſpect one of an o-ther, wherein there is a double compariſon made, and the greateſt common Diuiſor di-uideth it ſelfe alſo.

The nũbers giuen.

The compariſons.

The greateſt common di-uiſor.

The greateſt common diuiſor is found out as is afore ſaid. Now followeth the finding out of the leaſt number that may commonly be diuided by the numbers giuen: that is, ſuch a number, then which there cannot be a leſſe, which two or more numbers giuen may exactly diuide.

Euclide calleth it the leaſt that may be di-uided by certain numbers. For as many num-bers giuen may make many compound num-

bers,

bers, euen so also they may diuide them, out of which numbers to chose the least that may be diuided, is greatly auaileable to the readinesse and easinesse of numbring.

The numbers whose least common diuidend we seeke for, are either
- Only two alone, & those
  - Prime one to another. I.
  - or
  - Compound. II.
- Many, wherin we finde out the least number that may be diuided.
  - At the first. III.
  - Or
  - At the second time. IIII.

The finding out of the least number that may be commonly diuided by certaine numbers giuen, is learned by the Theoremes following.

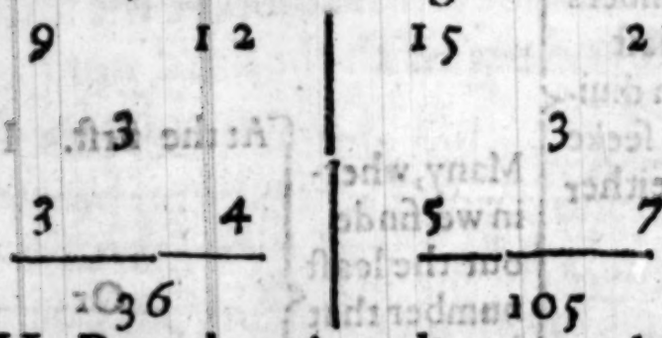
I. If the two numbers giuen, be prime one to an other, their product is the least number that may be commonly diuided by them.

As 15 the product of 3 and 5, being prime numbers one to an other, is their least cõmon diuidend. Item 63 is the least number commonly diuided by 9 and 7

II. But if the two numbers giuen, be compound one to an other, if the quotient of the one, found out by the greatest common diuisor, multiply the other, the product shall

be the leaſt number that may be diuided by them two. 36 prop. 7.

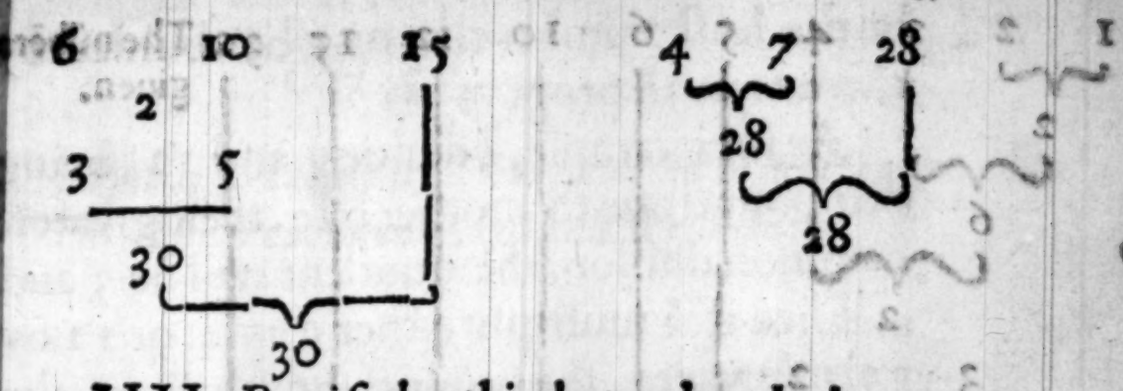
As for example, I diuide 9 and 12, being compound one to another by 3, their greateſt common diuiſor, the quotient will be 3 and 4. Then if I multiply either 9 by 4, or 12 by 3 croſſewayes, the product 36 ſhall be the leaſt number that may be commonly diuided by 9 and 12, the numbers giuen:



III. But when there be more then two numbers giuen, if the thirde diuideth that which the two former diuided, the firſt numbers found out ſhall be the leaſt that may commonly be diuided by them all.

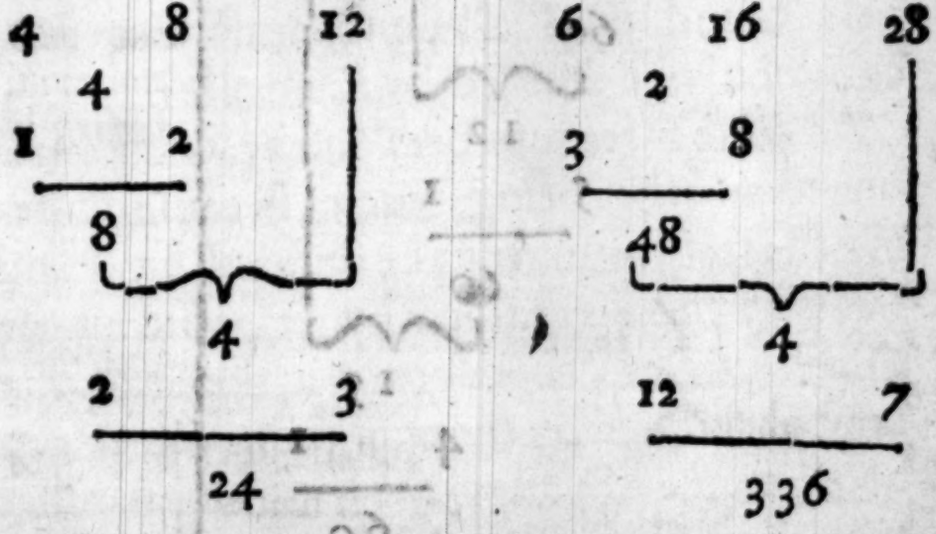
As theſe three numbers 6.10.15. compound one to an other being giuen, becauſe 30 being the leaſt number to be diuided by 6 and 10, may alſo be diuided by 15, which is the third number, therefore it is the leaſt that may be diuided by them all. Likewiſe in 4.7. 28, becauſe 28 being the leaſt number that may cõmonly be diuided by 4 and 7, it may alſo be diuided by the third number 28, as by it ſelfe, it ſhall be the leaſt number that may be diuided by the numbers giuen. As in theſe examples.

6 10

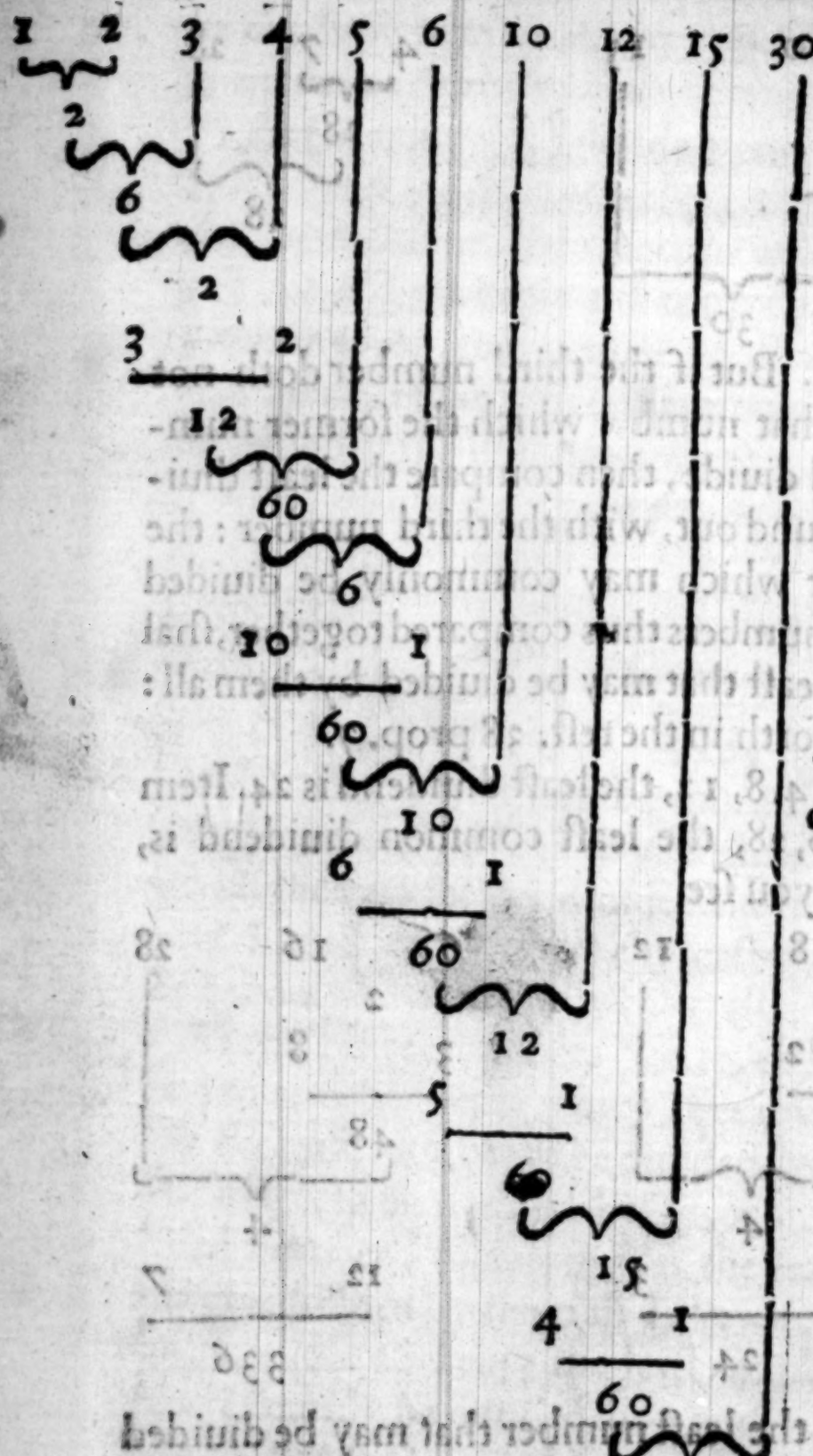


IIII. But if the third number doth not diuide that number which the former numbers did diuide, then compare the leaſt diuidend found out, with the third number: the number which may commonly be diuided by the numbers thus compared together, ſhal be the leaſt that may be diuided by them all: and ſo forth in the reſt. 38 prop. 7.

As in 4, 8, 12, the leaſt diuidend is 24. Item in 6, 16, 28, the leaſt common diuidend is, 336, as you ſee.

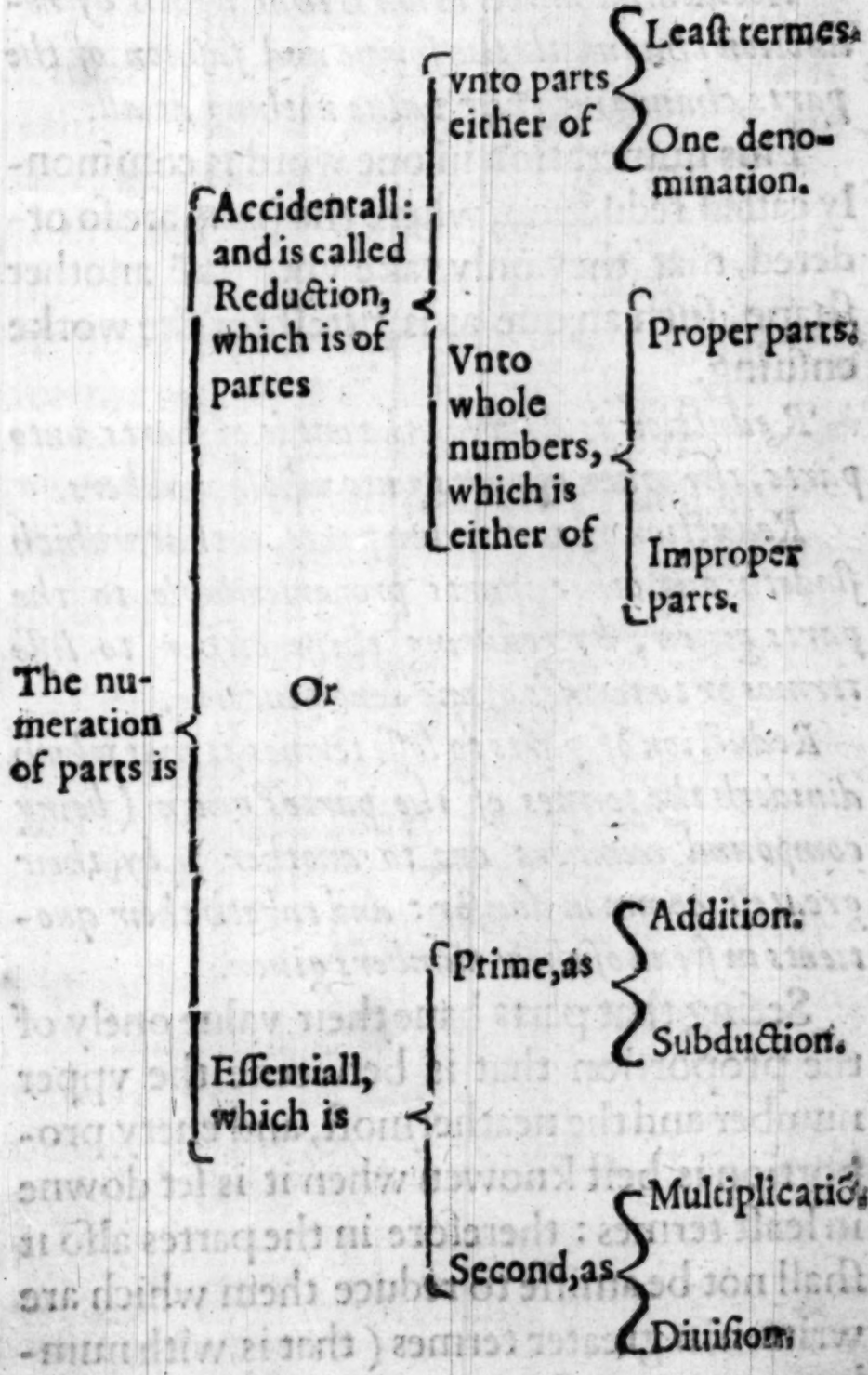


So, the leaſt number that may be diuided by 1, 2, 3, 4, 5, 6, 10, 12, 15, 30, is 60, whereupon it commeth to paſſe, that this number hath ſo greatly pleaſed the Aſtronomers in numbring their parts.


1 2 3 4 5 6 10 12 15 30
The nũber giuen.
2
6
2
3 2
12
60
6
10 1
60
10
The nũbers compared.
6 1
60
12
5 1
6
15
4 1
60
2 1
60
The leaſt cõmõ diuident

CHAP. VIII.

Of the Accidentall numeration of partes which commonly they call Reduction.



Hitherto we haue ſpoken of the numeration of vvhole numbers: now followeth the numeration of parts, wherein the numbers giuen are onely parts. And it is alſo double, Accidentall and Eſſentiall.

Accidentall numeration is that which by reduction chaungeth the forme and faſhion of the parts, chaunging their value nothing at all.

This numeration in one word is commonly called reduction, where the parts are ſo ordered, that they only take vnto thē another ſhape, ſuch an one as is fitteſt for the worke enſuing.

Reduction is double: the one is of parts vnto parts, the other of parts vnto whole numbers.

Reduction of parts vnto parts, is that vvhich findeth out other parts proportionable to the parts giuen, by reducing them either to leſſe termes or to termes of one denomination.

Reduction of parts to leſſe termes, is that which diuideth the termes of the partes giuen (being compound numbers one to another) by their greateſt common diuiſor: and taketh their quotients in ſtead of their numbers giuen.

Seeing that parts haue their value onely of the proportion that is betweene the vpper number and the neathermoſt, and euery proportion is beſt knowen when it is ſet downe in leaſt termes: therefore in the partes alſo it ſhall not be amiſſe to reduce them which are written in greater termes (that is, with numbers compound one to an other) to leſſer termes,

termes, the which thing is done by the common diuiſor. For the quotients haue the ſame proportion that the numbers diuided haue, as appeareth by the 35 prop. 7. So that their value is not chaunged.

The proofe alſo of this may be taken out of the 17 prop. 7. which ſaieth, *If a number multiply two other numbers, the products ſhall haue the ſame proportion that the multipliers haue.* Contrariwiſe therefore if a number diuide other two numbers, the quotients and the numbers diuided ſhall haue one proportion the one to the other. As if 4 multiply 6 and 8, their products 24 and 32 ſhall haue the ſame proportion which the numbers giuen haue. Againe, if you diuide theſe products 24 and 32 by 4, the ſame numbers to wit, 6 and 8 will returne againe in the quotient. If the parts giuen therefore be $\frac{6}{21}$ they muſt not be ſet downe in theſe termes, but muſt be reduced ſtraight wayes to $\frac{2}{7}$. Likewiſe $\frac{4}{16}$ diuided by 4, come to $\frac{1}{4}$. So $\frac{108}{252}$ may be reduced by 36 their greateſt common diuiſor to $\frac{3}{7}$.

By this kinde of Reduction not onely the termes of the ſame partes, but alſo the termes of diuerſe parts may be reduced croſſewayes one by an other: that is, the numerator of the one, and the denominator of the other. As $\frac{2}{3}$ and $\frac{5}{8}$ their termes being reduced croſſewayes will be $\frac{1}{3}$ $\frac{5}{4}$.

But this rule concerning the Reduction

of diuerſe partes, is not ſo generall as the other of one and the ſame parts. For it is more ſpeciall and peculiar, as it were to multiplication, as we ſhall ſee hereafter, wherefore we muſt not thinke that all ſuch partes giuen may be reduced after this manner.

Reduction of parts to one denomination is that, vvhich (vvhen the parts giuen are of diuerſe denomitations) taketh firſt the leaſt number vvhich may be diuided by the denominators, for the common Denominator, and then diuiding that number by the denominators of the parts giuen, ſetteth down the products made by the quotients, & by the numerators of the parts giuen for the numerators of the partswhich vve ſeeke for.

It is a thing of great vſe to reduce the partes giuen to one denomination, and to make them proportionable one to an other in the leaſt termes that may be: that is, of partes of diuerſe kindes, to make partes of one and the ſame kind. As for example if $\frac{3}{4}$ and $\frac{5}{6}$ were to be brought to one denomination, I muſt firſt ſeeke out the leaſt number that may be diuided by 4 and 6, which is 12, then I diuide 12 by their denominators 4 and 6, and multiply 3 and 5 the numerators of the parts giuen by the quotients 3 and 2, the products 9 and 10 ſhall be the numerators of the parts ſought out: and the leaſt number which might be diuided by the denominators: to wit, 12, ſhall be the common denominator to them both. As in this example.

$\frac{3}{4}$ $\frac{5}{6}$

$\frac{3}{4}$ $\frac{5}{6}$ the parts giuen $\frac{\cancel{12}}{\cancel{4}}$ (3 $\frac{3}{9}$ $\frac{5}{2}$ 10 the new numerators.

2 $\cancel{12}$

$\frac{2}{12}$ $\frac{3}{12}$ The cōmon denominator. $\frac{\cancel{12}}{\cancel{6}}$ (2 $\frac{9}{12}$ $\frac{10}{12}$ the parts found out.

$\cancel{12}$

See also the examples following.

$\frac{2}{3}$ $\frac{1}{8}$ $\frac{\cancel{24}}{\cancel{3}}$ (8 $\frac{8}{2}$ $\frac{1}{3}$

24 $\cancel{24}$ $\frac{16}{3}$

$\frac{\cancel{24}}{\cancel{8}}$ (3 $\frac{16}{24}$ $\frac{3}{24}$

$\cancel{24}$

Item

$\frac{3}{4}$ $\frac{5}{8}$ $\frac{7}{10}$ $\frac{\cancel{40}}{\cancel{44}}$ (10 $\frac{10}{3}$ $\frac{5}{5}$ $\frac{4}{7}$

8 $\frac{30}{25}$ $\frac{28}{28}$

2 $\frac{\cancel{40}}{\cancel{8}}$ (5

$\frac{4}{40}$ $\frac{5}{40}$ $\frac{\cancel{40}}{\cancel{40}}$ $\frac{30}{40}$ $\frac{25}{40}$ $\frac{28}{40}$

$\frac{\cancel{40}}{\cancel{10}}$ (4

9 10
18 20
$\frac{3}{4}$ $\frac{5}{6}$
24
12

16 3
$\frac{2}{3}$ $\frac{1}{8}$
24

In the first example therefore, the parts of one denomination are $\frac{9}{12}$ $\frac{10}{12}$, and they be likewise the least proportionable to the parts giuen. For as in $\frac{3}{4}$ 4 is to 12, so is 3 to 9: and in $\frac{5}{6}$ as 6 is to 12, so is 5 to 10, and so forth in the other examples.

By this rule of reduction we may knowe whether the parts being of diuers denominations be equal or vnequall. For being brought to one denomination, if they haue the same numerators they be equall: if not, they be vnequall: they that haue the greatest numerator are the greatest, and they that haue the least numerator are the lesse. As if it should fall into question what proportion there were betwene $\frac{5}{6}$ and $\frac{10}{12}$. Item betwene $\frac{5}{7}$ and $\frac{13}{21}$, whether they be equall or vnequall, this rule will take away the doubt: and will declare that the former parts are equal, the latter vnequal, and $\frac{5}{7}$ to be the greater, $\frac{13}{21}$ the lesser, as you see here.

$$\frac{5}{6} \quad \frac{10}{12} \qquad \frac{5}{7} \quad \frac{13}{21}$$

$$\frac{6}{1} \quad \frac{6}{2} \qquad \frac{7}{1} \quad \frac{7}{3}$$

$$\frac{12}{10} \quad \frac{12}{10} \qquad \frac{21}{15} \quad \frac{21}{13}$$

$$\frac{12}{12} \quad \frac{12}{12} \qquad \frac{21}{21} \quad \frac{21}{21}$$

To this reduction of partes to parts, succeedeth

deth the reduction of parts to vvhole numbers, and it is either of proper or improper parts.

The first reduction of proper parts taketh the numerators of many partes, hauing one denomination for the parts giuen.

As $\frac{3}{5}$ and $\frac{4}{5}$ are reduced to whole numbers when the numerators of the partes giuen 3 and 4, being as it were proportionable to the parts giuen, are taken for the parts themselues: so likewise we reduce these partes $\frac{1}{9}$ $\frac{4}{9}$ $\frac{7}{9}$ to whole numbers, when we take 1, 4, 7. the numerators for the parts themselues.

Reduction of improper parts taketh the quotient of the numerator diuided by the denominator, for the parts giuen.

This reduction is vsed when the partes are but one, and their value either equall or greater then the whole, as $\frac{3}{3}$ $\frac{10}{5}$ $\frac{27}{9}$. If you diuide the numerators of euery part seuerally by his denominator, the quotients will be 1, 2, 3.

Hitherto also may those parts be referred whose denominators doe not exactly diuide the numerators, as $\frac{36}{7}$ are reduced to $5\frac{1}{7}$ and $\frac{29}{6}$ to $4\frac{5}{6}$.

CHAP. IX.

Of the Essentiall numeration of parts.

VVE haue spoken *of the Accidentall numeration of partes. Now followeth the*

Essentiall numeration, which numbreth the parts either by encreasing, or diminishing them.

There be two kindes of Essentiall numeration. The first giueth the same denominator to the numerator found out, which the partes haue that be giuen: and it is either Addition or Subduction.

Addition addeth together the numerators hauing one denomination, and setteth the common denominator of the parts giuen vnder the totall.

Partes of diuerse denominations the one being greater or lesse then the other, can not be added together without confounding of the denominators, wherefore it is needfull to reduce them to one denomination. As if you adde $\frac{3}{5}$ to $\frac{4}{5}$ the totall will be $\frac{7}{5}$, or by reduction $1\frac{2}{5}$. Item adde $\frac{3}{4}$ to $\frac{5}{8}$, here if you should confound 4 with 8, the totall should haue his denominator of neither of them both. Wherfore the parts must first be brought to one denomination, and then be added together. As in this example.

The Reduction. The Addition. The totall.

6	5	6	11
3	5	5	8
4	8	11	Or by Reduction.
2	1		
8			$1\frac{3}{8}$

Likewise in adding $\frac{1}{2}$ vnto $\frac{5}{6}$ and $\frac{3}{11}$ there arise $\frac{106}{66}$, or by reduction, $1\frac{20}{33}$. The example is thus.

Reduction,

Reduction. Addition. The parts found out.

33 55 18 33

1 5 3 55 $\frac{105}{66}$

2 6 11 18 or els 1 $\frac{40}{66}$ which being

2 106 diuided by the cõmon

1 3 diuisor 2, are reduced

6 to $\frac{20}{33}$.

66

Subduction is a numeration vvhich ſubducting the numerators of the parts giuen, hauing one denomination, one from the other, taketh the common denominator of the partes giuen, for the denominator of the remainder.

As in ſubducting $\frac{3}{8}$ from $\frac{7}{8}$ there remaines $\frac{4}{8}$ or $\frac{1}{2}$. Item ſubduct $\frac{1}{3}$ from $\frac{10}{13}$. Here for ſo much as the parts be of diuerſe denominations, you muſt reduce them to $\frac{13}{39}$ and $\frac{30}{39}$, then ſubducting the numerators one from another, that is, 13 from 30, there remaines 17, vnder which you muſt ſet the common denominator, ſo that the partes found out are $\frac{17}{39}$. As in this example.

Reduction. Subduction. The parts found out.

13 30 30

1 10 13 17

3 13 17 39

13 3

39

Thus much of the first kind of the Essentiall numeration of parts: the second kinde giueth to the numerator found out an other denominator then the parts haue vvhich are giuen. And it is either Multiplication or Diuision.

Multiplication is that vvhich multiplying the termes of one kinde together (that is the numerators by the numerators, and the denominators by the denominators) taketh the products for the termes of the parts found out.

This multiplication is properly the finding out of other parts then are giuen, which are to one of the partes giuen, as the other is to an vnitie: and it is wrought by multiplying both the numerators and the denominators of the parts giuen one by an other, wherby we may necessarily conclude, that the partes found out are alwayes lesse then the parts multiplyed. As if there were $\frac{1}{2}$ to be multiplyed by $\frac{1}{3}$, multiplying the numerators and the denominators one by the other, the product is $\frac{1}{6}$, which is to $\frac{1}{3}$, as $\frac{1}{2}$ is to 1. For as $\frac{1}{2}$ is the halfe of one, so $\frac{1}{6}$ is the halfe of $\frac{1}{3}$, wherefore $\frac{1}{6}$ is lesse then $\frac{1}{3}$ or $\frac{1}{2}$. This perhappes may be perceiued more easily in the numbring of some certaine thing. As for example, in a degree there be 60 minutes, therefore $\frac{1}{2}$ of a degree are 30 minutes, $\frac{1}{3}$ 20 minutes, $\frac{1}{6}$ 10 minutes: as therefore 60 are to 30, so are 20 to 10. Againe in a shilling there are twelue pence, therefore $\frac{1}{2}$ of a shilling are sixe pence, $\frac{1}{3}$ foure pence, $\frac{1}{6}$ 2 pence:

as

as therefore a ſhilling is to ſixe pence, ſo are foure pence to two pence. Item if $\frac{5}{7}$ be multiplied by $\frac{3}{4}$ the product will be $\frac{15}{28}$.

The abridgement of multiplication.

If the oppoſite termes of the partes giuen be numbers compound one to an other, then in their ſtead take their quotients found out by their greateſt common diuiſor: and then worke the multiplication as before.

As, multiply $\frac{3}{4}$ by $\frac{5}{18}$. Here for ſo much as 3 and 18 being oppoſite termes, are compound one to another, you ſhall ſet downe in their ſtead their quotients 1 and 6 found out by their common Diuiſor, ſo that the parts thus reduced ſhall be $\frac{1}{4}$ and $\frac{5}{6}$. Now multiplying 1 by 5, and 4 by 6, the product will be $\frac{5}{24}$. Likewiſe if $\frac{3}{7}$ ſhould be multiplyed by $\frac{14}{15}$, firſt you muſt reduce them to $\frac{1}{1}$ and $\frac{2}{5}$, which being multiplied one by the other, make $\frac{2}{5}$. As in theſe examples.

5

$\frac{1}{4}$ $\frac{5}{6}$

24

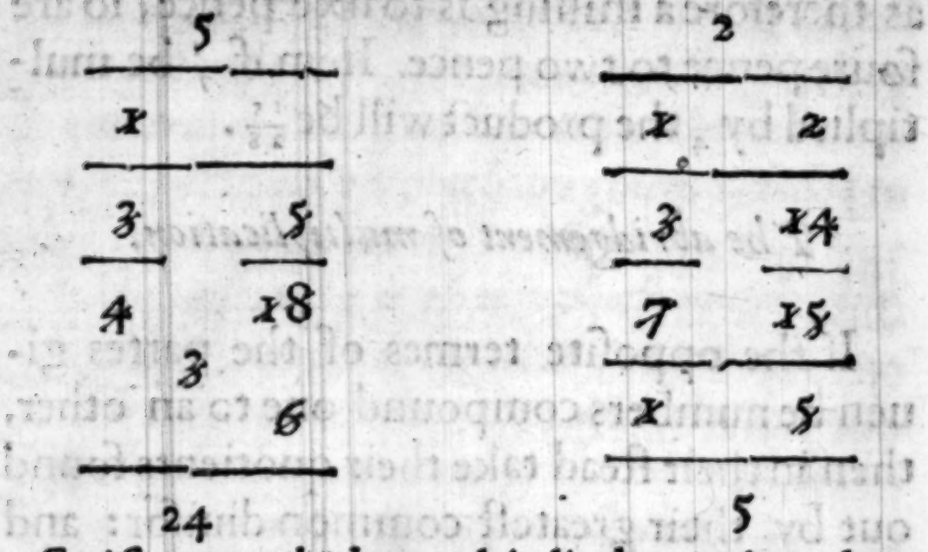
2

1 2

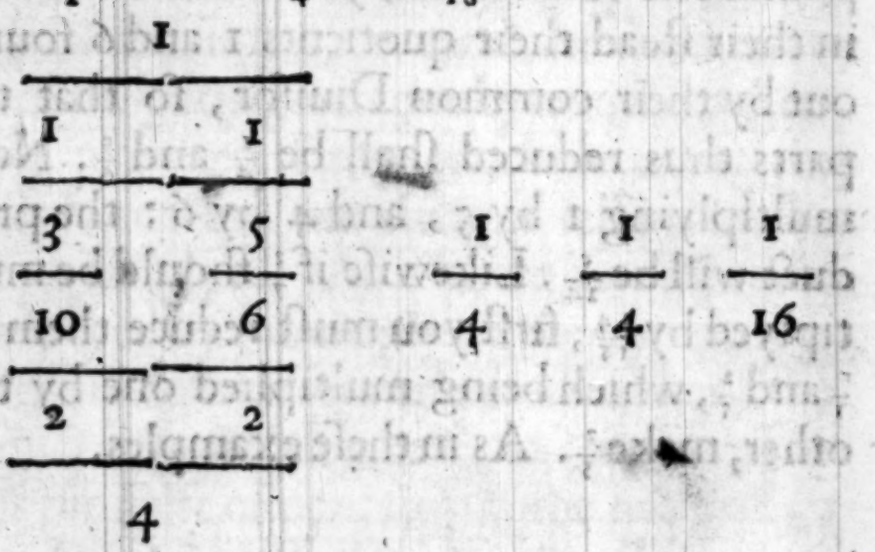
$\frac{3}{7}$ $\frac{14}{15}$

1 5

5



So if $\frac{3}{10}$ $\frac{5}{6}$ and $\frac{1}{4}$ be multiplied together, they giue $\frac{1}{16}$. For first $\frac{3}{10}$ and $\frac{5}{6}$ being multiplied together, doe make $\frac{1}{4}$, which being afterwarde multiplied with $\frac{1}{4}$ make $\frac{1}{16}$. Thus:



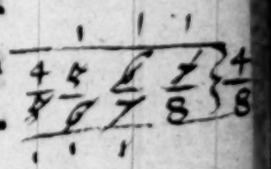
Correlarie gathered of the former abridgement.

Of this reduction of numbers this consequence is gathered. If the crosse or opposite termes of many parts be equall one to the other, then neglecting them which are equall, the numerator remayning set ouer the denominator remaining, is the product of thē all.

As

As if $\frac{4}{5}\ \frac{5}{6}\ \frac{6}{7}\ \frac{7}{8}$, were to be multiplied together, I caſt away the middle numbers which are equall, and ſee 4 the numerator ouer 8 the denominator, ſo that the product is $\frac{4}{8}$ or by reduction $\frac{1}{2}$. The reaſon hereof is this, becauſe the croſſe termes being as it were compound one to the other, are by reduction to one denominator brought to vnities thus, $\frac{4}{1}\ \frac{1}{1}\ \frac{1}{1}\ \frac{1}{8}$. But for ſo much as an vnitie in multiplication chaungeth not a number, therefore the figure 4 multiplied by the three vnities in the ſame ranke, maketh but 4, no more doth 8.

$\frac{4}{5}\ \frac{5}{6}\ \frac{6}{7}\ \frac{7}{8}\ \Big\}\ \frac{4}{8}$



Moreouer it appeareth by this proportion which is in multiplication, that ſeconde parts may well be reduced to principall parts by this meanes, as $\frac{1}{3}$ out of $\frac{3}{4}$ are $\frac{3}{12}$ or $\frac{1}{4}$ of the whole, which thing may be eaſily perceiued in the numbring of any thing, as $\frac{3}{4}$ of a degree are 45 minutes, whoſe third part are 15 minutes, which are equall to $\frac{1}{4}$ of a degree. So $\frac{2}{3}$ of $\frac{3}{4}$ parts of a crowne are halfe a crowne. For $\frac{3}{4}$ are three ſhillings nine pence, whoſe $\frac{2}{3}$ parts are two ſhillings ſixe pence. So $\frac{5}{6}\ \frac{1}{2}$ of $\frac{3}{8}$ by multiplication are brought to $\frac{15}{96}$ or $\frac{5}{32}$. And thus much of multiplication of parts.

To reduce fraçions of fraçions into one ſingle fraçion

Diuiſion is that vvhich in ſtead of the partes giuen diuideth vvhole numbers proportionable vnto them.

The diuiſion of parts is properly the finding out of a quotient which is in proportion to the diuidend, as an vnitie is to the diuiſor.

Whereby it is manifeſt, that the quotient muſt needes be greater then the parts which are diuided, otherwiſe then it is in diuiſion of whole numbers: for there the quotient is alwayes leſſe then the diuidend. This inuention commeth of the diuiſion of ſuch numbers as are proportionable to the parts giuen, for ſo much as the partes themſelues can not be diuided.

Firſt therefore diuide theſe partes of one denomination, to wit, $\frac{9}{10}$ by $\frac{3}{10}$. Here therefore you ſhall reduce them (by the former reduction to whole numbers) vnto 9 and 3, then ſhall you diuide 9 by 3, the quotient will be 3, which declareth that $\frac{3}{10}$ the diuiſor is contayned in $\frac{9}{10}$, the diuidend three times, and as $\frac{3}{10}$ are to one, ſo are $\frac{9}{10}$ to three: the which quotient therefore is bigger then the partes diuided.

Diuide $\frac{5}{6}$ by $\frac{3}{6}$, that is, ſee how many times $\frac{3}{6}$ are contayned in $\frac{5}{6}$. Firſt you ſhall reduce the partes giuen to whole numbers proportionable vnto them, that is, to 5 and 3, then ſhall you diuide 5 by 3, the quotient will be $1\frac{2}{3}$. Contrariwiſe, $\frac{3}{6}$ diuided by $\frac{5}{6}$ giue in the quotient $\frac{3}{5}$. For both the denominators being neglected, I ſet the one numerator ouer the other as parts, becauſe the one cannot be diuided by the other.

Furthermore if you diuide parts of diuerſe denominations, as $\frac{3}{4}$ by $\frac{1}{3}$, you ſhall firſt make the partes of one denomination, reducing them

them to $\frac{9}{12}$ and $\frac{4}{12}$: then by the former reduction you shall bring them to whole numbers, and diuide 9 by 4, the quotient will be $2\frac{1}{4}$. Diuide $\frac{8}{9}$ by $\frac{5}{6}$, reduce the partes to one denomination, that is, to $\frac{16}{18}$ and $\frac{15}{18}$, then reduce these partes hauing one denomination to whole numbers 16 and 15, which being diuided one by another, make in their quotient $1\frac{1}{15}$.

16 15

$\frac{8}{9}$ by $\frac{5}{6}$

3 3 2

18

CHAP. X.

Of mixt numeration.

- Mixt numeration is either
  - Accidentall, and is reduction of
    - Whole numbers,
    - Or
    - Mixt numbers.
  - Essentiall, wherof there are two kinds
    - Prime, as
      - Addition.
      - Subduction.
    - Second, as
      - Multiplicatiō.
      - Diuision.

or thus

48 45

$\frac{8}{9}$ by $\frac{5}{6}$

3

48

45 (1 $\frac{1}{15}$

HEtherto of ſimple numeration. It remayneth now to ſpeake of mixt numeration vvherein the numbers giuen are whole numbers and parts together. And this numeration alſo as that of the partes is double. Firſt Accidentall, which beſide the former reduction conſiſteth of the reduction both of vvhole and mixt numbers vnto parts.

Reduction of vvhole numbers vnto parts taketh the product made of the vvhole numbers giuen, & the denominator of the parts giuen, for the numerator of the parts which we ſeeke for.

This reduction may be drawen out of the reduction of improper parts, as it were a conſequence following it. For as you reduce $\frac{4}{4}$ by diuiſion vnto 1 whole, ſo contrariwiſe you ſhall bring one whole to 4 quarters, if you multiply one whole by 4 which is the denominator of the parts, for they will return to $\frac{4}{4}$.

Item, as $\frac{30}{5}$ by diuiſion are reduced to 6 an whole number: ſo if you multiply 6 by 5 the denominator of the partes, and take the product 30 for the numerator, and 5 for the denominator, there will be againe $\frac{30}{5}$ which are equall to the whole numbers giuen. Thus whole numbers as often as the vſe of numeration requireth, are written as parts, by ſetting an vnitie vnderneath them. As for example theſe whole numbers, 2, 5, 9, hauing an vnitie ſet vnderneath them for their denominator ſhall be as it were partes after this manner, $\frac{259}{111}$.

Reduction of mixt numbers taketh the product made of the vvhole numbers giuen, and the denominator of the partes giuen being added to the numerator, for the numerator of the partes found out.

As you may reduce $4\frac{3}{7}$ into partes, if you multiply 4 by 7, the denominator of the parts giuen, and adde 3 the numerator, to 28 the product, taking the totall 31 for the numerator, and setting the denominator giuen vnderneath it, thus, $\frac{31}{7}$. Likewise reducing $6\frac{3}{4}$, there come forth $\frac{27}{4}$, and so foorth in the rest.

Thus much concerning the accidentall numeration of mixt numbers: now remaine the speciall kindes of numeration, vvhereof the first numbreth the termes of mixt numbers seuerally by themselues alone: that is, the vvhole numbers by themselues, and the parts by themselues, and that by adding or subducting them, vvhereupon it is called addition and subduction.

These kinds of numeration differ nothing from the former, but are in proportion all together like vnto them.

The examples of Addition.

I. Adde 3 and $2\frac{4}{5}$ the totall will be $5\frac{4}{5}$.

II. Item adde $\frac{3}{4}$ to $6\frac{6}{7}$, first the parts added together are $\frac{45}{28}$, or by reduction $1\frac{17}{28}$, the totall therefore is $7\frac{17}{28}$.

III. Adde $8\frac{7}{9}$ to $4\frac{1}{2}$, first the parts added together are $\frac{23}{18}$, or by reduction $1\frac{5}{18}$, then adde, 1 to 4 make 5, which being added to 8, are 13. So that the totall is $13\frac{5}{18}$.

The examples of Subduction.

I. Take from 3 the whole number $\frac{5}{9}$, here I reduce an vnitie of the whole number to $\frac{9}{9}$, and there remaines 2. Afterward I ſubduct $\frac{5}{9}$ out of $\frac{9}{9}$, there remaine $\frac{4}{9}$. The remainder therefore of the numbers giuen to be numbred is, $2\ \frac{4}{9}$.

II. Item, take $\frac{1}{4}$ from $8\ \frac{1}{5}$. Here becauſe $\frac{1}{4}$ cannot be taken out of $\frac{1}{5}$, to wit, the greater from the leſſe: I take 1 from 8, there remaine 7, then I bring 1 and $\frac{1}{5}$ by the reduction of mixt numbers vnto $\frac{6}{5}$, from whence (after I haue brought the parts to one denomination) I take $\frac{1}{4}$, there remaine $\frac{19}{20}$. So that the remainder of the numbers giuen is, $7\ \frac{19}{20}$.

III. Take 4 from $10\ \frac{1}{2}$, the remainder will be $6\ \frac{1}{2}$.

IIII. Take $5\ \frac{7}{12}$ from $7\ \frac{5}{8}$. Here firſt you ſhall make the partes of one denomination, that is, $\frac{15}{24}\ \frac{14}{24}$. Then taking 5 from 7. there remaine 2, then alſo ſubducting the partes hauing one denomination one from the other, there remaines $\frac{1}{24}$. The remainder therefore of the parts giuen is, $2\ \frac{1}{24}$.

The latter kinde of mixt numeration numbreth the termes of the mixt numbers giuen ioyntly together: that is, the vvhole numbers vvith the partes, and that by multiplying or diuiding them: vvhereupon it is called Multiplication or Diuiſion.

If $\frac{3}{4}$ were to be multiplyed by 5, ſetting downe the whole numbers after the manner

of

of parts, I multiply $\frac{5}{1}$ by $\frac{3}{4}$, the product is $\frac{15}{4}$ or by reduction $3\frac{3}{4}$.

The value of the parts is sought out after this manner. As if you would knowe what $\frac{5}{8}$ of a circle were, I set downe the whole partes of a circle, that is, 360 degrees after the common order of partes, thus, $\frac{360}{1}$: then multiplying the nūbers giuen together, the product is $\frac{1800}{8}$, or by reduction vnto whole nūbers, 225. So many degrees are $\frac{5}{8}$ of a circle.

So likewise if you would haue $\frac{1}{7}$ of $10\frac{5}{8}$, first you shal reduce the last of the two numbers giuen by the reduction of mixt numbers into $\frac{85}{8}$, which being multiplyed by $\frac{1}{7}$, make $\frac{85}{56}$, or by reduction $1\frac{27}{56}$.

Likewise if both the numbers giuen be mixt, as $4\frac{1}{3}$ and $5\frac{3}{1}\frac{1}{1}$, the product made by multiplication will be $\frac{754}{33}$, or by the reduction of improper parts, $22\frac{28}{33}$.

The examples of Diuision.

If 4 were to be diuided by $\frac{7}{9}$, you must find out whole numbers proportionable to the numbers giuen: for the whole 4 being reduced into 9, the parts will be $\frac{36}{9}$ and $\frac{7}{9}$, wherby I see that the whole numbers 36 and 7, are proportionable vnto the numbers giuen: and they being diuided one by the other, make in the quotient $5\frac{1}{7}$.

Item if $\frac{1}{2}$ be diuided by the whole number 3, 3 being reduced vnto $\frac{6}{2}$, I finde the whole numbers 1 and 6 to be proportionable to the numbers giuen: which being

diuided the one by the other, make in the quotient $\frac{1}{6}$. Item, diuide $8\frac{2}{3}$ by $4\frac{3}{5}$: here reduce the termes of both numbers by the reduction of mixt numbers to $\frac{26}{3}$ and $\frac{23}{5}$, then make them of one denomination thus, $\frac{130}{15}$ $\frac{69}{15}$, afterwards reduce them to whole numbers proportionable vnto them, that is, to 130 and 69, which being diuided one by the other, the quotient shall be $1\frac{61}{69}$.

The end of the first part.

THE SECOND PART OF ARITHMETICKE.

CHAP. XI.

Of the kindes of proportionalitie or reason.

E. The second part of Arithmeticke consisteth in the cõparison.

- Either of the termes, and is called proportionality or reason, whereof
  - The one is in equall termes: and is called the proportionalitie or reason of equalitie.
  - The other is in vnequall termes: & is called the proportionalitie or reason of inequalitie, and is to be considered
    - Either according to the difference of the termes.
      - Either prime, as
        - Multiplex.
        - Or
        - Simple, which is
          - Muitiplex Superparticular.
          - Or
          - Multiplex Superpartiens.
      - Of the greater inequality, & that is
      - Of the lesser inequality, which is to be diuided into as many kinds as the former is diuided into.
    - Or according to their quotient: and herein we are to note,
      - The kinds: for of the proportionalitie of inequalitie some is,
        - Or
        - The handling of the kinds wherin we are to cõsider their
          - Notation.
          - Numeration, which is,
            - Additiõ.
            - Or
            - Subduction.
- Or of the proportionalities or reasons. F.

Hitherto of Simple, now followeth Compare Arithmeticke, vvhich consisteth in repres sing the nature and the qualitie of termes compared together.

In writing of proportions the numbers whereby they are expressed, are by a Geometricall phrase called terms: by which word are to be vnderstood such quantities as are cõpared together. Hereupõ generally such things as are compared together are called Termes.

The cõparing of termes together, is called Proportion, which is a certain respect which the terms haue one to another according to their quantitie.

This definition in this place is more general then the 3 def. of the 5 book of Euclide, which

Proportion

saith, that *Proportion is a certaine respect of two magnitudes of one kinde onely, according to their quantitie.* For in that place it was requisit that there shold be mention made of magnitudes of one kind only, because that magnitudes are not al of one kind. But here those words may be omitted, because al nũbers are of one kind, & euery one may be compared together: and moreouer the analogie or respect which they haue one to another, may be expressed by an other number, which thing cannot be done in all magnitudes, though they be all of one kind irrationall, and surde magnitudes can not be expressed by number.

The comparison of termes, is either equall or vnequall, whereupon it is called Proportion of equalitie or vnequalitie.

Proportion of equalitie is one only, and not diuided into many kinds. Equalitie

As the proportion of 5 to 5, of 6 to 6, of 12 to 12. For both in these, and in all other equall termes, the respect is all one, neither hath it any more kindes, because that one and the same number cannot be either more or lesse equall to another.

Proportion of inequality is knowne either by the difference of the termes, or by their quotient. Inequalitie

The proportion of difference, is that whereby one terme differeth from another.

The proportion of the quotient is when one terme contayneth another.

The proportion of difference is known by subduction, the proportion of the quotient is knowne by diuision.

As in the naturall order of the numbers the difference betwene 1 and 2, 2 and 3, 3 and 4, 4 and 5, 5 and 6, &c. is all one, that is an vnitie. Againe, the difference betweene 8 and 5 is 3, betweene 6 and 4 it is 2. But the proportion of the quotient is by an especiall phrase simply called proportion. As, the proportion of 2 to 1 is double, of 3 to 2 sesquialter, of 24 to 3 octuple. For the former termes (which cõmonly are called the antecedents) being diuided by the termes following (which are called the cõsequents) the quotients are 1, 1 $\frac{1}{2}$, 8: whereby we may gather that there may be now and then the same difference betwene the termes though as touching Sesquialt

the quotient their proportion differ, as in 4 8 and 12, the difference betwene 4 and 8, and betwene 8 and 12, is all one, yet the proportion gathered by their quotients is diuerse, for in the one it is 2, double, in the other it is but sesquialter $1\frac{1}{2}$. Contrariwise also in termes the proportion of the quotient may be all one, though their difference be diuerse: as in 3 and 9, 4 and 12, the quotient is 3 triple in them both: yet the difference is not a like, for in the former it is 6, in the second it is 8.

Sesquialter

Proportion of inequalitie is double, for either the greater terme is antecedent to the lesse, or consequent. Hereupon the one is called the proportion of the greater inequalitie, the other the proportion of the lesser inequalitie.

Proportion of ye greater & lesser inequalitie

When two termes are compared together, the one is called the Antecedent, the other the consequent. Therefore, as often as the greater terme is the antecedent and goeth before, it is called the greater inequalitie. As when 6 is compared to 3, and 3 to 2. But if the lesser terme be the Antecedent, and we consider what difference there is betweene that and the greater, it is called the lesser inequalitie.

Both these fornamed proportions are either Prime or Seconde.

Prime, is a proportion of one kinde onely, and is either simple or multiplex.

Simple proportion, is when the greater terme

contayneth

contayneth the lesse once and somewhat more, as proportion Superparticular and Superpartiens.

Superparticular

Superparticular, is a simple proportion wherein the greater terme contayneth the lesse, and one part more of the lesse.

As when 3 is compared to 2, 6 to 5, 8 to 7. For in these proportions the antecedent contayneth the consequent once, and one part beside of the lesser terme. As it may be perceiued by their quotients when they be diuided thus, $1\frac{1}{2}$, $1\frac{1}{5}$, $1\frac{1}{7}$. This kinde contayneth vnder it infinite other kindes, whereof the first is sesquialter, the second sesquitertia, the fourth sesquiquarta.

Item there is the same proportion betwene 12 and 8, that is betwene 15 and 10, as appeareth by their termes reduced to the least denomination thus, $1\frac{1}{2}$.

Superpartiens

Superpartiens is a simple proportion, wherein the greater terme contayneth the lesse, and certaine parts of it.

As 5 to 2, 7 to 4, 13 to 8. For the antecedents being diuided by the consequents, their quotients declare their proportion on this manner, $1\frac{2}{3}$, $1\frac{3}{4}$, $1\frac{5}{8}$. Likewise the proportion betwene 15 and 9 is superpartiens, as appeareth by the quotient and the parts reduced to the least denomination. $1\frac{2}{3}$.

Multiplex

Multiplex is a prime proportion vvhen the greater terme contayneth the lesse certaine times exactly.

Multiplex is defined after the same man-

ner in the ſecond defin. of the fifth booke of Euclide, which ſaith that *Multiplex is a greater quantitie in reſpect of a leſſe, when the leſſe meaſureth the greater.*

As the proportions of 4 to 2, 9 to 3, 16 to 4 are multiplices, that is to ſay, double, triple, quadruple: for their quotients are 2, 3, 4.

Second proportion conſiſteth of two kindes of proportion, and is either multiplex ſuperparticularis, or multiplex ſuperpartiens.

It ſeemeth that theſe kinds of proportions are made of the former, by ioyning two of their names together. For the one is the third kinde of proportion ioyned to the firſt, and the other is the third ioyned to the ſecond.

multiplex ſuperparticular

Multiplex ſuperparticularis is a compound proportion, wherein one terme containeth another certaine times, and one part beſides.

As 5 to 2, 10 to 3, 13 to 4. For the quotients declaring their proportions, are $2\frac{1}{2}$, $3\frac{1}{3}$, $3\frac{1}{4}$.

multiplex ſuperpartiens

Multiplex ſuperpartiens, is vvhen the one terme containeth the other, & beſides more parts then one.

Here we are to vnderſtande, that they be the true partes which are not equall to the leſſer terme, as in 11 to 4, 12 to 5, 22 to 6. For the quotients are $2\frac{3}{4}$, $2\frac{2}{5}$, $3\frac{2}{3}$.

CHAP.

CHAP. XII.

Of Notation and Numeration of proportions.

THus much for the kinds of proportion, now followeth the handling and practise of them: which consisteth in Numeration and Notation.

Notation is the writing of the antecedents aboue, and the consequents beneath.

The Antecedents	4	8	6	5	1
			Or		
The Consequents	3	3	2	6	2

The proportions being written, are expressed by the quotient (the parts if there be any, being reduced to the least termes) which declareth as well their proportion as their denomination.

In multiplex proportion it is called double, triple, quadruple, octuple proportion, where the quotients are 2, 3, 4, 8. In proportions of lesser inequalitie this prepositon *Sub* is generally added in them all. As for example, it is called in Submultiplex, Subduple, Subtriple, Subquadruple, &c. In Superparticular proportion (becauſe the halfe of the greater number is the proportion) it is called after the manner of the Latines *Sesquialtera*, *Sesquitertia*, *&c.* putting the worde *Sesqui* in the beginning, and the denominator of the partes in the ending as when the quotients are $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$. In proportions of the lesser inequalitie we say, *Subsesquialtera*, *Subsesquitertia*, *&c.*

Likewiſe Submultiplex, Superparticular, Submultiplex Superpartiens. For in compound proportions the names are alſo compound, as in $3\frac{1}{1}$ triple ſeſquiquarta, $2\frac{4}{4}$ dupla ſuperquadripartiens, &c.

And thus much for inſtruction ſake. For I know that theſe wordes are very ſtraunge to vs, and not heard of, nor vſed among the beſt writers. For they when they would ſpeake of the proportions of things, were wont to expreſſe them by two numbers. As Archimedes in expreſſing the proportion of the circumference of a circle to the Diameter, ſayd not that it was triple ſeſquitertia, but as 22 was to 7.

Moreouer the quotients teach vs the way not onely how to expreſſe the proportions giuen, but alſo how to ſet them downe in the leaſt termes. For if you ſet an vnitie vnderneath, or ouer againſt the quotients being whole numbers, you ſhall find out the leaſt termes of the multiplex which you ſeeke for. As in the multiplices, if you ſet an vnitie right againſt 3,4,5, the proportion in the leaſt termes will be triple, quadruple, quintuple. Againe, if you reduce the quotients being mixt numbers, by the ſeconde reduction of mixt numbers vnto partes, you ſhall in like manner finde out the leaſt termes in the other kind of proportions: as in Superparticular proportion, if you ſeeke for the leaſt termes of $1\frac{1}{2}$ ſeſquialtera, $1\frac{1}{3}$ ſeſquitertia,

tertia, $1\frac{1}{8}$ ſeſquioctaua, then the quotients reduced after the foreſaide manner, the leaſt termes will be theſe.

3	4	9
2	3	8

The like is in all the other kinds, as if you would find out the leaſt termes in duple ſuperpartiens fiue ſixe parts, then ſhall you of $2\frac{5}{6}$ make the antecedent 17, and the conſequent 6, and they ſhall be the leaſt termes in the proportion aſſigned, and ſo foorth in the reſt.

Thus much for notation. Numeration is the addition or ſubtraction of proportions.

Addition, is that which taketh the products made by multiplying the like termes together, (that is, by multiplying the antecedents by the antecedents, and the conſequents by the conſequents) for the termes of the compound proportion.

The adding of proportions is ſet downe in the fifth def. of the ſixth booke thus. *A Proportion is ſaid to be made of proportions, whē the quātities of the proportions multiplyed together produce another proportion.* By the quantities of the proportions he meaneth the quotients whereof the proportions take their names. As if you adde a ſeſquialtera to an equall proportion, it maketh a ſeſquialtera. Adde a ſeſquiquarta to a duple ſuperpartiens two third parts, the product will be a ſeſquitertia: as you ſee in theſe examples.

	Simple propor.	Cõpound proport.	Simple.	Cõpound.
Anteced.	3 4	12	1 8	40
Conseq.	2 4	8	4 3	12
That is, adde	1 $\frac{1}{2}$ vnto 0	1 $\frac{1}{2}$	1 $\frac{1}{4}$ vnto 2 $\frac{2}{3}$	3 $\frac{1}{3}$

That the products made of multiplying the termes together, haue a compound proportion appeareth by the 5 prop. of the eight booke of Euclide, which saith, that *Plaine or superficiall numbers, are in that proportion one to the other, which is composed of their sides.* As 3 and 12 being plaine numbers, haue proportion one to another, according to their sides, as you see.

The sides.		The plaine nũbers.
3	4	12
		3
1	3	

If diuerse proportions be to be added together, first adde two of them, then adde the compound to that which remaineth. As if the proportions giuen were,

2	6	10
1	5	4

First two of them added together, make $\frac{12}{5}$ to which the third proportion being added, maketh a Sextuple. $\frac{120}{5}$: 6.

Thus euery proportion may be tripled or quadrupled: if the termes of the proportion giuen being set downe three or foure times

times be added together after the foresaide manner. So a double proportion being tripled, maketh an octuple. And a sesquialtera being doubled foure times, maketh a quintuple sesquidecima, as you see here.

2	2	2	8	Also	3	3	3	3	81	$(5\frac{1}{16}$
1	1	1	1		2	2	2	2	16	

Subduction of proportions, is that which taketh the products made of the opposite termes of the proportion giuen, for the termes of the proportion remaining.

Which proportion is least, and which may be subduced one from the other in Multiplices may be easily knowne by the denomination onely. But in the other (the termes being compared together as partes) it may be gathered by that which hath bene taught before in the partes. Subduction seemeth to be the consequence of that which was set downe in the fifth def. of the sixt booke. For if the addition of proportions be wrought by multiplycation, it followeth then that subduction must be wrought by diuision. Therefore whether you diuide the quotients one by the other, or multiply the opposite termes of the partes giuen one by the other (for in partes the multiplying of the termes crosseways is as much as diuision) the termes produced will haue a proportion left like vnto the former.

As take a ſeſquialtera from a double proportion, the remainder will be a ſeſquitertia. Item, take a ſeſquiquarta from a double ſeſquialtera, there remaineth a double, as in theſe examples.

	The prop. giuen.	The rem.	The propor. giuen.	The remainder.
Anteced.	2 3	4	5 5	20 2 or
Conſeq.	1 2	3	2 4	10 1
	2 from 1 $\frac{1}{2}$	1 $\frac{1}{3}$	2 $\frac{1}{2}$ from 1 $\frac{1}{4}$	2

There is no multiplication in proportions properly as in numbers, vnleſſe you take the continuing of the proportions for their multiplication. Neither is there any diuiſion required in them beſide the finding out of one or more proportionals. Wherefore we will proceede to the reſt.

CHAP.

CHAP. XIII.

Of proportion and the kinds thereof, but especially of Arithmeticall proportion.

F. The comparison of reasons, is

- Either of vnequall reasons, and is called improportion, whereof in the 8 d. 5. with the which Arithmeticke doth not meddle.
- Or of equall reasons, and is called proportiõ, consisting
  - Either in the equalitie of the differẽces: and is called Arithmeticall proportiõ, which is
    - Either disiunct,
    - Or continuall, and this againe is,
      - Either simple,
        - Continuall, as they follow one another.
        - Or
        - Interrupted, when we seeke but for certaine termes.
      - Or multiplex, & is called progression: wherin we are to obserue,
        - Either the order of the termes,
        - Or the summe of the termes.
  - Or in the equalitie of the quotients: and is called Geometricall proportion. G.

NOw that vve haue ſpoken of comparing the termes together it remayneth to ſpeake of the compariſon of reaſons, vvherein firſt vve haue to deale vvith proportion.

Proportion is the equalitie of reaſons.

Euclide in the fourth definition of the fifth booke, 'defineth proportion to bee a ſimilitude of reaſons. Ariſtotle in the fifth booke and 3. Chapter of the Ethickes, calleth it an equalitie of reaſons. If wee ſhoulde define it to be the Identitie of reaſons, we ſhould not varie much from the meaning of Euclide, who in the ſixth definition of the fifth booke ſpeaketh after the ſame manner, calling them *proportionall quantities vvhich haue the ſame reaſon one to another*. And albeit that theſe definitions doe properly agree to Geometricall proportion, yet if you take the word reaſon in the largeſt ſenſe, for any kinde of reſpect it ſhall be generall, ſo that as reaſon or proportionalitie is a mutuall reſpect of two termes one to another, ſo proportion ſhall be a reſpect of like termes one to another.

Proportion is either Arithmeticall or Geometricall.

Arithmeticall proportion is an equalitie of differences.

As in 8, 7, 6, and in 12, 9, 6, 3. Here are two proportions, wherein you may ſee an equalitie of differences, or an equall exceſſe

ceſſe of the antecedents. In the firſt, the difference is 1, in the latter it is 3. So likewiſe in 15, 11 : 8, 4, as 15 exceede 11 by 4, ſo 8 exceede 4 by 4.

Arithmeticall proportion is double, for either it is continued in the termes, or it is ſeuered, vvhereupon it is called continuall, or diſiunct proportion.

Theſe differences are common both to Arithmeticall, and to Geometricall proportion. That proportion is continuall wherein not onely the extremes are continued together by one meane, but generally euerie proportion, wherein many and ſundrie middle termes haue a continuall reſpect one to another, ſo that any of them may be eyther the antecedent or the conſequent. In ſo much, that not onely the proportion betwene 3, 2, 1 is continuall: where 2 is the conſequent of the former, and the antecedent of the latter proportion: but this alſo 5, 7, 9, 11, where the ſame difference is continued from the firſt to the laſt. And any of the middle numbers either 7 or 9, haue the ſame nature in reſpect of them that goe before and follow after, that 2 had in the former example.

Proportion is ſeuered in the termes when the firſt onely hath the ſame reſpect to the ſecond, that the third hath to the fourth, but the ſecond agreeth not proportionablie with the thirde: that is,

when all the termes in order as they followe are not of like proportion one to the other. As 10, 8, 4, 2, are ſeuerally proportionall becauſe 2 the firſt difference is not continued betwene 8 and 4. So that here is a ſeuering not onely of the termes, but alſo of the proportions. Leauing therefore in this place to ſpeake of diſiunct proportion, wee will handle that which is continuall.

Continuall proportion is either Simple, where the extreames haue but one middle terme: or manifold, wherein many middle termes go on in order continually: whereupon it is called progreſſion.

Progreſſion is a continuall enlarging of the termes of the proportion giuen: wherein there are two extreames giuen, and manie other numbers in the middle betwene them.

In Arithmeticall progreſſion we are to finde out two things, either the number of the termes as they ſtand in order, or elſe their ſumme.

Sometimes the termes be all ſought for as they follow in order, and are found out by the continuall adding of the difference to the laſt terme in the progreſſion.

As if you would know the termes of this progreſſion 1, 5, 9, 13, as they follow in order, you muſt adde 4, which is the difference vnto 13, then the fifth terme will be 17, to which if you adde 4 againe, it maketh 21, which is the ſixth terme, and ſo foorth infinitely.

Sometime

Sometime breaking off their naturall order, we seeke for some one terme in the progression, which may be found out by the Theoreme following.

The progression increasing, if you take an vnitie from the name of the terme which you seeke for, & adde the product made of the remainder, and the common difference of the termes, vnto the first terme, the totall will be that which you desire. The progression decreasing, if that product be taken frō the first terme the remainder is the terme sought for.

As in this progression 4, 7, 10, where the difference is 3, I desire to know the 12 terme beginning at 4. Here 12 is the name of the terme which I seeke for, from whence I take 1, and multiply 11 which remaine by 3 the difference, the product is 33, whereto I adde 4 the first terme. So the totall is 37 the terme which I sought for. Likewise the 30 terme is 91, and so foorth.

The termes	4	7	10	37	91
The names or number of the termes.	1	2	3	12	30

Thus much for the finding out of the termes. The finding out of the summe is the halfe of the product made by multiplying the two extremes added together, and the name of the last terme, as it standeth in order.

As in this example, 1, 5, 9, 13, 17, 21, both the extreames added together make 22, the

name of the last terme is 6, for it is the sixt in order. Whereby if you multiply 22, the product will be 132, the halfe whereof (that is) 66, is the summe sought for.

This rule may be proued by the 41 prop. of the first booke of Euclide, which saith, *If a parallelogramme and a triangle haue one and the selfe same base, and be in the same altitude, the parallelogramme shall be double to the triangle.* The numbers then of the progression represent the triangle: whereof one side is the name of the last terme, or (as they commonly terme it) the number of the places: the other is the extremes added together. But for so much as the product made of these two sides maketh not a triangle, but a parallelogramme, therefore in taking the one halfe I gather by it how bigge the platforme of the triangle is, which is the summe of the progression.

The example of this progression 2, 4, 6, 8, 10, is set downe here as you see:

	111111111100	The whole paralle-
	111111110000	logram is 60, wher-
5	111111000000	fore 30 which is the
	111100000000	halfe, is the summe
	110000000000	of the progression.
	12	

To set forth the profite and vse of this in resoluing certaine questions, let this be the first example.

For

For so much as a naturall day hath 24 houres, I demaund how many strokes the clocke vvill strike in a day.

In this example we seeke for the number of the strokes beginning at one, and adding one till you come to the 24 terme. After this manner the clockes in Italy and Norinberge are made. Both the extremes added together make 25, which being multiplyed by 24, the name of the last place, make 600, whose halfe are 300 strokes, which is the summe sought for.

Another.

There be 30 egges laide in a rancke, euery one three foote from another. I must put them in a basket vvhich standeth 13 foote from the first egge: yet so that at euery time I goe to and fro I must take vp but one. The question is, how farre a man may go before I haue gathered them all.

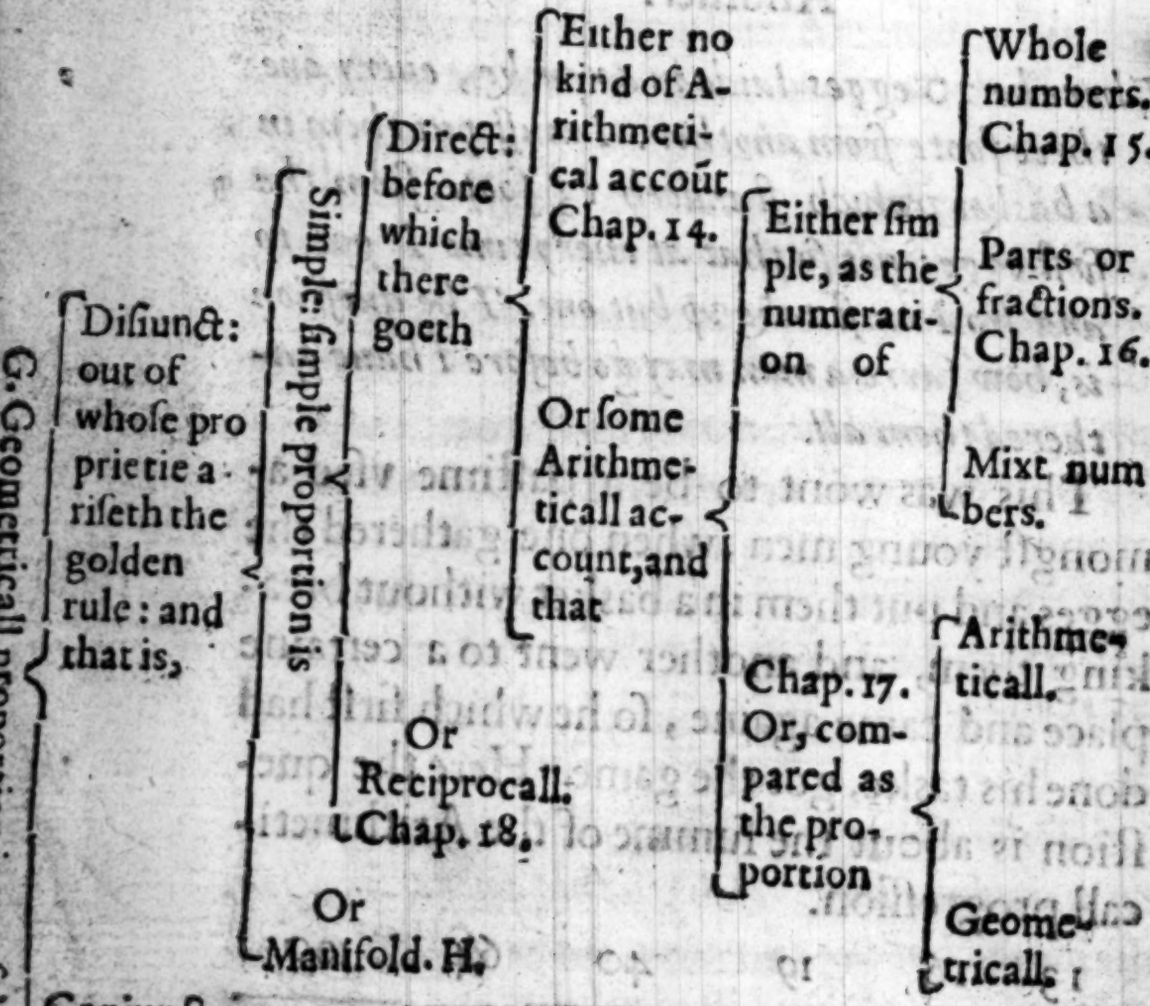
This was wont to be a pastime vsed amongst young men, when one gathered the egges and put them in a basket without breaking them, and another went to a certaine place and came againe, so he which first had done his taske, got the game. Here the question is about the summe of the Arithmeticall progression.

13	16	19	40	64	100
1	2	3	10	18	30

Both the extremes together are 113, the product made of this ſumme, and the name of the laſt terme is 3390, whoſe halfe is 1695 feete, which make 339 Geometricall paces. This ſpace he goeth and commeth, wherefore the double of it which was firſt found out, that is 3390 feete, or 678 paces is the ſum which we ſought for.

CHAP. XIIII.

Of Geometricall proportion, and of the golden rule belonging thereunto.



Thus

THus much of Arithmeticall proportion, Geometricall followeth, vvhich is the equalitie of reasons.

Here this word Reason is not taken generally, but specially, for the equalitie of the quotients arising of the numbers compared together. As in 2, 6, 4, 12, what reason or proportionalitie is betweene 2 and 6, the same is betweene 4 and 12, that is, subtriple. And if foure numbers be directly proportional, they shall be also proportionall backwarde, and crossewayes in this manner.

2	6	4	12	Direct.
6	2	12	4	Backward.
2	4	6	12	Crossewaies.

Geometricall proportion is double: Disiunct, or Continuall.

Disiunct is that that agreeth neither in termes nor proportionalitie, or reason.

As in 6, 3, 16, 8, for here are two double proportions distinquished both in termes and in reason, because that betweene 3 and 16 there is not the same reason observed, that is betweene 6 and 3 and betweene 16 and 8.

Disiunct proportion consisteth of foure termes at the least. It hath this propertie that the product of the extremes, is equall to the product made of the middle numbers, and contrariwise. 16. p. 6. & 19. p. 7.

Among the properties of Geometricall proportion ſet foorth by Euclide, this is moſt excellent and neceſſarie to be knowne, becauſe the force and fruitfulneſſe thereof is ſuch, that it may very well be called the head of cyphering, that is, the foundation and off-ſpring of all rules, as we ſhall ſee hereafter. It hath this meaning, that the number made by multiplying of the firſt into the fourth, is equall to the number made by multiplying the ſeconde into the thirde, as in 6, 4, 18, 12, the product made of 6, multiplyed by 12 is 72, the which number ariſeth by multiplying the middle numbers 4 and 18 one by another. And this thing is proper to continuall, and generally to all Geometricall proportion, as wee ſhall ſee hereafter.

This propertie miniſtreth vnto vs the golden Rule, vvhich three proportionall numbers being giuen, diuiding the product made by multiplying the third into the ſecond by that vvhich remaineth, findeth out the fourth proportionall number.

Out of the fore named propertie, as out of a moſt plentifull ſpring is drawen this Rule, wherein conſiſteth the chiefeſt partes of caſting accounts. The which ſome for the excellencie of it doe call the Rule, others the rule of Proportion, other the rule of three, and all men all moſt in generall, for the excellent vſe of it, doe call it the golden Rule.

The

The drift and end of it, is the finding out of a fourth terme, or number, that ſhall haue the ſame proportion with the three numbers giuen: the which thing it bringeth to paſſe by multiplying two termes of the proportions giuen one by the other, that is, the antecedent of the one, and the conſequent of the other, and diuiding the product by the other which remaineth: for the quotient will be the fourth number of the proportion giuen.

As in 3, 2, 9, I deſire to knowe the fourth that ſhall be to 9, as 2 to 3, multiply therefore 2 by 9, and diuide 18 the product by 3 which remaineth, the quotient will be 6, the proportionall number which was ſought for.

The cauſe of this conſequence is manifeſt, becauſe hauing the product of the middle numbers, it is all one as if you ſhould ſay, that you had the product made of the firſt and fourth numbers, and to proue that, or to bring to light the fourth number lying hidde in the product, you muſt diuide the product by the firſt number, and ſay, that the quotient is the other number vnknowen which made the diuidend: becauſe that euerie number is diuided by thoſe numbers by which it is made, and euery product diuided by one of the multiplicators bringeth forth the other.

As in 9, 6, 12, I deſire to finde the fourth, multiply therefore 6 by 12, that is the middle

number, then I say, the product 72 is the product arising of 9, and the fourth number vnknowen. The product then being had, and one of the multipliers knowen, the other cannot any longer be hid. Wherefore I diuide 72 by 9 the quotient will be 8, which is the other multiplier, and also the fourth number in proportion: thus, 9, 6, 12, 8.

This rule is vsed in proportion simple or manifolde.

Simple proportion is that which consisteth onely of foure termes or numbers.

The forme vvhereof is double, for either the proportion proceedeth directly forward, or else backewarde, vvherevpon it is called Direct, or Reciprocall.

I call that Direct proportion, vvherein as the first terme is to the second, so the third is to the fourth, and crossewayes.

Which is Direct proportion, may be easily gathered by this, that by howe much the third number is greater or lesse, by so much the fourth in order shall be greater or lesse, whether the numbers be giuen ordinarily, or crosslewaies, or backwards.

And it is handled two manner of wayes, for sometimes besides the ordering of the terms there is nothing else required: sometimes it requireth some former kind of numeration.

The ordering or disposing of the termes to conclude the proportion attributeth to euery one that is giuen his owne place: that is, to the thing

vvhich

vvhich is called into queſtion, it giueth the third place: the firſt place to the thing of the ſame kind: the middle place to that vvhich remaineth.

The lawfull order of the termes is that in diſtinguiſhing the reaſons, we ioyne the conſequent of euery reaſon to his owne antecedent of the ſame kind. As if I ſhould ſay, ſeeing Iupiter in foure moneths moueth tenne degrees in the Zodiacke: therefore in ſixe monethes it moueth 15 degrees. Here the proportion ordinarily ſhall ſtand thus, 4, 6, 10, 15. For as 4 monethes is to 6 monethes, ſo is 10 degrees vnto 15 degrees. For here alſo the product made of the extreames is equall to the product made of the middle numbers.

But the termes in the Rule of proportions is otherwiſe ordered, becauſe in things of one kind the queſtion is not ſo moued, that they ſhould be ioyned together, but croſſewayes. For we doe not moue the queſtion cuſtomably from the antecedent of the ſecond reaſon to his conſequent thus, if 4 moneths are 6 monethes, what are tenne degrees? But the queſtion is moued from the conſequent of the firſt reaſon to the conſequent of the ſecond croſſewayes: if 4 monethes giue 10 degrees, then 6 monethes ſhall giue 15 degrees. Therefore ſeeing the proportion is moued croſſewayes: and thereby may be handled more conueniently, and may be better conceiued by this meanes of the learner,

therefore in the ordering of the termes, we place them which be of like ſort croſſe-wayes in ſundrie places, not ioyntly together.

And hereupon the queſtion being moued ſomewhat confuſedly, we muſt ſet that terme that hath the queſtion adioyned vnto it in the third place, and the terme of the ſame kinde, that is the terme which betokeneth the ſame thing, and is called by the ſame name in the firſt place: and the other in the middeſt: then muſt we followe the Methode and direction of the rule, multiplying the thirde by the ſecond, or contrariwiſe, the ſecond by the third, and diuiding the product by the firſt.

The examples.

A traueller is to goe 180 *miles, vvhereof he goeth* 9 *in two dayes, the queſtion is vvhen he ſhall end his iourney.*

Here the queſtion beeing propounded ſomewhat confuſedly, you ſet in order the termes. And for ſo much as the queſtion is moued about the 180 miles, in what time they may be gone thorough, you ſhall ſet that number in the third place, the 9 miles being a terme of the ſame ſort, ſhall ſtande in the firſt place, the two dayes in the middle. Then multiply 2 by 180, and diuide the product 360 by 9, the quotient wil be 40,

the

the fourth number in proportion anſwering the queſtion. The example is thus:

If 9 require 2 dayes, ergo 180,40 *daies.*

If a pole ſixe foote long ſtanding vpright vppon the ground caſteth a ſhadowe tenne foote long, I demaund how high that Tower is, vvhich at the ſame time caſteth a ſhadowe 125 *foote long.*

Here I ſee that the numbers giuen are proportionall. For as the height of the pole being knowen is vnto the ſhadowe, ſo is the hight of the tower vnknowen vnto his ſhadow. The termes therefore being ſet duely in order, and handled according vnto the rule, the example ſhall be thus.

If 10 *giue* 6: *ergo* 125 *ſhall giue* 75.

After the ſame manner in this rule may the parts be ordered and handled, as may be perceiued by the examples following:

If the ſixt part of the Moone encreaſeth vvhen ſhe is diſtant from the Sunne the twelfth part of a circle: how bigge ſhall ſhe be when ſhe is diſtant from him halfe a circle.

Here the firſt terme ſhall be $\frac{1}{12}$ of a circle, the ſecond $\frac{1}{6}$ of the Moone, which in that diſtance is lightened with the light of the Sun: the third $\frac{1}{2}$, the worke being wrought there ariſeth 1, which ſignifieth that the Moone is at the full when ſhe is halfe a circle off from the ſunne. The termes are thus:

$\frac{1}{12}$ $\frac{1}{6}$ ergo $\frac{1}{2}$ 1.

I ſpend in the fourth part of a moneth $\frac{5}{6}$ of a crowne, vvhat doe I ſpend in $\frac{7}{10}$ of a moneth?

Here the termes are giuen in order, wherfore I multiply the third by the ſecond, the facit will be $\frac{7}{12}$, which being diuided by $\frac{1}{4}$ after the manner of parts or fractions, the quotient will be 2 $\frac{1}{3}$, ſo that the example ſhall be thus:

2 140
$\frac{1}{4}$ X $\frac{5}{6}$ — $\frac{7}{10}$
60

$\frac{1}{4}$ $\frac{5}{6}$: ergo $\frac{7}{10}$ 2 $\frac{1}{3}$.

But here if you deſire to knowe the value of $\frac{1}{3}$ of a crowne or of any other fraction what ſo euer, you ſhall ſearch it out in like manner by the rule. For that they teach commonly, that the number made by multiplying of the numerator, and the partes of the whole being knowen, muſt be diuided by the denominator, whereuppon proceedeth the value of the parts giuen, it is nothing elſe but to practiſe the rule of proportion. For it is all one, as if you ſhould ſet the denominator of the parts in the firſt place, the numerator in the third, and the parts of the whole being knowen in the middlemoſt place, and ſo worke accordingly. As, put caſe three parts of a crowne be 45 pence, you ſhall find out the value of $\frac{1}{3}$ part thus:

If 3 be 45 pence: 1 ſhall be 15 pence.

If one or more of the termes be mixt, you muſt vſe the ſame methode which before was vſed

vsed in whole and broken numbers: sauing that both the whole and broken numbers must first be reduced into fractions. As the question being moued, two yardes and $\frac{1}{6}$ of cloth are bought for fiue crownes, what shall I pay for $5\frac{3}{4}$ of a yarde: you shall reduce therfore the first terme vnto $\frac{13}{6}$, the second vnto $\frac{5}{1}$, the third vnto $\frac{23}{4}$, whereof there amount 13 crownes & $\frac{7}{26}$, which is the number required.

690

$\frac{13}{6} \times \frac{5}{1} \times \frac{23}{4}$ ($13\frac{7}{26}$

52

Certaine abridgements.

In whole numbers it shall not be amisse to haue regarde vnto the abridgement following. If of the proportionall numbers giuen, the first and second, or the first and thirde be compound one to another, then they being diuided by the greatest common diuisor: the proportion shall be more briefly concluded by the prime numbers.

The reason of this abridgement is this, because the termes being chaunged after this manner, yet the proportion is not chaunged by the 15 prop. of the fifth booke, where it is saide, that *Like parts of multiplices, and also their multiplices compared together haue one and the same proportion.* As in 12, 32, 15, because 12 and 32 are compound one vnto the other, therfore they being diuided by 4, I take their quotients 3 and 8 in stead of the numbers giuen. Thus, if 3 giue 8, therefore 15 shall giue 40.

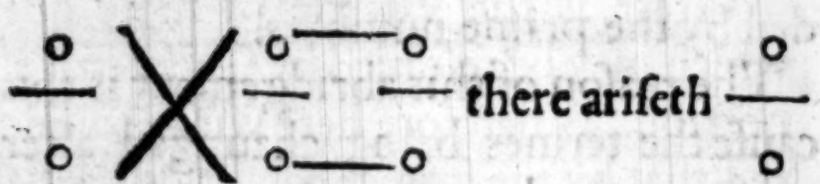
Likewise seeing the first and the third that

is to say 12 and 15 are also compound the one vnto the other: taking their quotient 4 and 5, being prime numbers found out by their greatest common Diuisor, for the numbers giuen, the proportion may be easily concluded in this manner.

If 4 *be* 32: *therefore* 5 *shall be* 40.

In the fractions it seemeth a briefe kind of working to dispatch the matter by multiplication onely: to wit, if you take the number made by multiplying of the denominator of the first terme, by the numerator of the second terme, and then by the numerator of the third terme, for the numerator of the fourth: and the *facit* of the numerator of the first terme, by the denominators of the second, and third for the denominator of the 4.

The draught of them is thus:



As if $\frac{1}{4}$ giue $\frac{2}{3}$, what shall $\frac{5}{6}$ giue? Multiply 4 by 2, there ariseth 8, then by 5, the product is 40, which is the numerator of the number which we seeke for. Furthermore multiply 1 by 3, and then by 6, the *facit* will be 18, the denominator of the fourth terme. The example is thus:

$\frac{1}{4}$ $\frac{2}{3}$ ergo $\frac{5}{6}$ $\frac{40}{18}$, or 2 $\frac{2}{9}$.

CHAP.

CHAP. XV.

Of examples of the rule of proportion, requiring some kinde of simple numeration before them.

IN the former examples, the rule of proportion was simple.

But oftentimes it requireth some certaine kind of numeration to goe before it: the vvhich numeration serueth to the ordering of the termes of the principall proportion, according to the nature and lawes of the queſtion propounded, that therby the proportionall number, vvhich vve deſire to know, might be vvith more eaſily found out.

And this numeration is drawen out of both the partes of Arithmeticke: Sometime out of abſolute or ſimple Arithmeticke, and is either ſimple or mixt.

Great is the varietie of thoſe queſtions which may be reſolued by Geometricall proportion. Wherefore here wit is to be required, that by the former conditions and nature of the queſtion offered, wee may be able to perceiue what is meete for the vnknitting and concluſion of the proportion, and howe alſo (as it were a certaine preparatiue to the future demonſtration) the termes of the proportion lying hid, are to be vnfolded, ordered, ſet downe, and to be applyed to the rule it ſelfe.

Firſt therefore in this example addition of vvhole numbers is required.

A Vintener buying at Ansa in Italie 120 *hogesheads of wine for* 2400 *ſs. laid out for the cariage of them to Baſill* 1200 *ſs. and to the waggeners for lading and vnlading them* 30 *ſs. for cuſtome in diuerſe places he paide* 50: *his expenſes in his iourney came to* 64. *He deſireth to gaine* 800. *I demaund for vvhat he ſhall ſell an hogshed.*

2400
1200
. . 30
. . 50
. . 64
. 800
4544 ſ.

In this example Addition ordereth one of the proportionall termes: for adde the price to his expences, and to the gaine, the totall will be 4544. Whereby I perceiue that the number of the hogsheds aſſigned is vnto this totall ſumme as one hogshed is to the price which we ſeeke for. Wherefore the example ſhall be thus.

Hog.	*ſſs.*	*Hog.*	*ſſs.*
If 120 *be worth*	4544 : *ergo* 1 *giues*		37 $\frac{15}{15}$

Another.

Two trauellers going from Baſill vnto two contrarie places, doe trauaile a diuerſe paſe one faſter then the other: the one goeth euerie day fiue, the other but three miles: I demaund how many miles they ſhall be aſunder at nine dayes end.

Here alſo Addition helpeth to the ordering of one of the termes. For adde their daily iourneyes 5 and 3, the totall will be 8, which is the diſtance for one day, whereupon the rule concludeth,

day.

day.	miles.	dayes	miles.
1	giueth 8: therefore	9	72

Examples of ſubduction going before.

A pipe of vvater voydeth into a ceſterne containing 250 firkins, euery houre 24 firkins, now there runne out at another pipe euerie houre 16 firkins, I demaund in vvhat time the Ceſterne will be filled.

Here you muſt find out by ſubduction, one of the termes of the proportion to be ordered according vnto the rule. For reaſon telleth me that I muſt ſeeke out the exceſſe of the filling aboue the emptying. Subducting therfore 16 firkins from 24, there remaines 8 firkins, for ſo many euery houre are left in the ceſterne. Hereby the proportion may eaſily be concluded:

8 *Remaine in* 1 : *ergo* 250 31 $\frac{1}{4}$

A certaine man ſelling an hundred pound of certaine kinde of marchandiſe for 15 *crownes, found at the length that in an hundred crowns he had loſt ſo much as he paide for euerie hundred vvaight: I demaund vvhat he paid for an hundred.*

Here you ſhall ſubduct the price of an hundred pound, to wit, 15 crownes, (which number alſo repreſenteth the loſſe) out of an hundred crownes, the remainder will be 85, the firſt terme of the proportion. For euen as 85 crownes which is the ſtocke diminiſhed, is vnto an 100, which is the whole ſtocke:

euen ſo 15 the price of an hundred waight which was loſt, muſt be vnto the ſtocke whereof it was rayſed: whereupon the termes ſhall ſtand thus: If there remaine

Crownes.

85 *of* 100: *ergo* 15 *de* 17 $\frac{11}{17}$.

Examples vvherein Multiplication goeth before.

A Polonian going vnto the Vniuerſitie of Baſill to ſtudie, deliuereth vnto a Marchant 126 *golden Ducats (euery one being valued at* 28 *vrſati, to receiue againe of him for them at Baſill ſiluerlings (vvhich commonly are called Thaleri) being vvorth* 18 *vrſati. The queſtion is how many ſiluerlings he ſhall receiue.*

28 vrſati makes a ducat

18 vrſati maks a ſiluering or Thaleri

15 vrſati maks a crowne

Here you ſhall reduce by multiplication 126 ducats vnto vrſati, accounting to euerie ducate 28, whereof the whole ſumme amounteth vnto 3528 vrſati. Then looke what proportion there is betweene 18 vrſati, and one ſiluerling, the ſame there is betweene the whole number of the vrſati, and betwene the ſiluerlings which we ſeeke for. Wherefore the termes ſhall be thus:

Vrſati *Thaler.*

18 *are vvorth* 1: *ergo* 3528, 196

This and ſuch like examples may be alſo reſolued by the rule of reciprocall proportion, as ſhall appeare hereafter.

He that hath 34 *crownes for his yearely vvages, vvhat hath he for eight dayes?*

Here

Here I see there is a question of proportion moued, but for so much as the first and third termes be of a diuerse denomination, I reduce them by multiplication to one kind of denomination. And so generally the terms of diuerse kindes are by a former multiplication to be drawen to the least denomination. Therefore I take for a yeare 365 dayes, and that the account might be the more exact, I diuide the 34 crownes into 510 vrsati (reckoning 15 vrsati to a crowne) or if it please you to bring them into a smaller coyne, you may resolue them into pence, and then the example shall be thus:

365 510: *ergo* 8 11 $\frac{13}{73}$

Sometimes Multiplication goeth before with Addition or Subduction.

The examples of the first.

He that desireth in sixe crownes to gaine 10 *vrsati, how shall he sell a pound of that vvare vvhich cost* 20 *vrsati?*

For so much as the money here set downe is of a diuerse denomination: first of all I reduce the 6 crownes by multiplication vnto vrsati: valuing euery crowne at 15 vrsati, the *facit* is 90. Whereunto if I adde the gaine, the whole summe of the vrsati, to wit, 100 shall be the middle terme of the proportion after this manner:

If of 90 there arise 100: then of 20 22 $\frac{2}{9}$

In the least termes thus:

9 10: *ergo* 20 22 $\frac{2}{9}$

A Butcher deliuering vnto his ſeruant 216 crownes, ſent him to market willing him to buy as many Oxen, Calues, Weathers, and Hogges of euery one an euen number, as that money vvould afford him to buy: I demaund how many beaſts he bought.

Firſt let vs appoint the prices: an Oxe at 11 crownes, a Calfe at two, a Weather at one crowne and 5 vrſati, an Hogge at 3 crownes and tenne vrſati. Here then is there a double multiplication with addition required, to the ordering out of the principall proportion. For here you muſt reſolue the 216 crownes by 15 (for ſo many vrſati doe Germaines commonly value their crownes) into 3240 vrſati: And likewiſe reduce the ſeuerall prices of euerie beaſt vnto one denomination: to wit, the prices of the Oxen vnto 165, of the Calues vnto 30, of the Weathers vnto 20, of the Hogges vuto 55 vrſati, and then adde together the prices of the ſame denomination, the totall 270 ſhall be the firſt terme of the proportion. For as this number is to one of each kind of beaſts, ſo the whole ſumme of money is vnto the number of the beaſtes vnknowen. The example is thus:

270 *giue* 1: *ergo* 3240, 12

The example of the latter.

A theefe hauing by ſtealth conuayed away certaine ſiluer veſſels, in his flight goeth euerie day

day 128 furlongs. Foure dayes after the owner following the theefe on horſebacke, rideth euerie day 174 furlongs. I demaund vvhen he ſhall ouertake him.

For the reſoluing of this queſtion, two termes of the principall proportion muſt be ſet downe, the one by multiplication, the other by ſubduction. For firſt we muſt ſee how farre the theefe was gone before the owner eſpying the matter entred his iorney. Wherfore I multiply 128 furlongs by 4 dayes, the *facit* is 512 furlongs. Secondly, I conſider how much the latter outgoeth the former euery day. Subducting therefore 128 from 174 the remainder is 46. Then looke what proportion the dayly exceſſe hath vnto a day, the ſame proportion ſhall there be betweene the former flight and all thoſe dayes wherein the latter ouertaketh the former. So that the termes of the proportion ſhall be thus:

46 *giue* 1 : *ergo* 512 11 $\frac{4}{23}$.

CHAP. XVI.

Of examples of the golden Rule, vvherein there is required ſome numeration of broken or mixt numbers.

IN the former examples before the proportions could be concluded, there vvas required a certaine numeration of vvhole numbers: in thoſe vvhich follow, the numeration is of broken or mixt numbers.

Example. 1.

A certaine man ſelling his horſe for 75 crownes, findeth he hath gotten the fourth part of the money vvhich he paide for him, I demaunde vvhat he paide for him?

Here adde vnto the money which he gaue (which is ſuppoſed to be $\frac{4}{4}$) the gaines, to wit $\frac{1}{4}$, the totall will be $\frac{5}{4}$: for he that of his ſtocke laide out raiſeth $\frac{1}{4}$, he by 4 gaineth 5. Wherefore the principall proportion ſhal be thus: 5 75: *ergo* 4, 60

A Farmer hauing gotten by one yeares harueſt certaine ſackes of corne, payeth to his Landlords for his yeares rent, to one $\frac{2}{5}$, *to another* $\frac{1}{4}$, *and hath remaining* 35 *ſackes: I demaund how many he had at the firſt.*

In this and ſuch like examples of broken numbers hauing diuers denominations, out of the leaſt number which may commonly be diuided by the denominators of the fractions giuen, muſt be drawen whole nũbers being vnto the number diuided as the parts aſſigned do require. As here the leaſt number which may be diuided by the denominators ſet downe which are 5 & 4, is 20: wherof $\frac{2}{5}$ are 8, & $\frac{1}{4}$ is 5, which being added together, & the cõmon denominator ſet vnderneath do make $\frac{13}{20}$. The farmer then of his whole heap of corn which is ſuppoſed to be 20, paid 13, wherefore $\frac{13}{20}$ being ſubducted out of $\frac{20}{20}$, there remaine $\frac{7}{20}$: wherupon the denominators being omitted, the termes ſhall thus be ſet together:

7 are

7 *are* 35 : *ergo* 13 *were* 65

A certaine man selling his horse lost the sixth part and $\frac{2}{15}$ *of the money which he paid for it, and had remaining* 243 *crownes: the question is vvhat the horse cost him.*

This example is like to the former: for you must take the least number which may commonly be diuided by the denominators of the parts giuen, 6 and 15, to wit, 30: whose parts being of the same denomination, to wit, $\frac{1}{6}$ is 5, and $\frac{2}{15}$ are 4. The partes added make $\frac{9}{30}$, which being subducted from $\frac{30}{30}$ leaue $\frac{21}{30}$ where vpon the proportionall numbers according to the rule shall be:

21 *are* 243 : *ergo* 30 347 $\frac{1}{3}$

So much was the principall stocke wherfore he lost 104 crownes $\frac{1}{7}$.

Sometimes not onely simple, but also mixt numeration helpeth oftentimes to the ordering of proportion.

For example.

Of three siluer pots, the first wayeth 12 *halfe ounces and* $\frac{2}{3}$, *the second* 16 $\frac{1}{2}$, *the third* 23 $\frac{1}{6}$. *The halfe ounce is worth* 11 *vrsati* $\frac{1}{2}$: *vvhat then is the price of the pots?*

Here of necessity must addition of mixt numbers goe before, although the principall proportion also consisteth of mixt numbers. For the waight of all the pots being added together, you shal haue halfe ounces 52 $\frac{1}{3}$: wherupon the proportion shall be thus framed.

If 1 *cost* 11 $\frac{1}{2}$, *therefore* 52 $\frac{1}{3}$ 601 $\frac{5}{6}$

A Vintener hauing bought 16 *hogsheds of wine for* 27 $\frac{3}{4}$ *crownes, the price of the vvine falling, could not ſell them but for crownes* 19 $\frac{5}{6}$: *I demaund vvhat he ſhould haue loſt if there had bene* 84 *hogsheds?*

Here ſubduction of mixt numbers muſt go before. For the price of the thing ſold for leſſe then it coſt being ſubducted out of the price which it was bought for, will bring out the ſecond terme of the proportion, ſo that you may eaſily anſwere the queſtion. Therefore 19 $\frac{5}{7}$ being ſubducted from 27 $\frac{3}{4}$ there remaines 7 $\frac{11}{12}$, the loſſe in 16 hogſheads of wine. Wherefore the termes of proportion ſhall thus be ſet downe:

In 16 *hogſ. there are loſt* 7 $\frac{11}{12}$ *: ergo* 84, 41 $\frac{9}{16}$

Mixt numeration is more common a great deale then any other, becauſe the meaſures, prices, &c. of things are ſeldome contayned in whole numbers: wherefore the ſtudious may here haue plentifull matter to exerciſe him ſelfe in.

CHAP. XVII.

Of examples of the golden Rule requiring proportion before them.

Hitherto numeration vſed in ſimple and abſolute Arithmeticke ſerued to the finding out of the concluſion in Geometricall proportion.

Yet

Yet oftentimes ſome part of compared Arithmetick is required therunto: as for example, proportion either Arithmeticall or Geometricall. Firſt then let theſe be examples wherein Arithmeticall proportion muſt go before.

An Eſpie enformeth his Captaine of the diſtance of his enemies campe after this manner. If we take our iourney hence, and goe the firſt day 30, the ſecond 28, the third 26 furlongs, diminiſhing after that order euery day two furlongs, vve ſhall meete vvith them at ſifteene dayes end. Now the enemie knowing of their comming, made toward them, ſo that at nine dayes end they met together. I demaund how farre the Captaine vvent before he met vvith the enemie.

Here firſt and formoſt we muſt gather the ſum of the proportion Arithmeticall (which commonly we call progreſſion) whoſe termes are 15, and the extremes 30, and 2: the whole ſumme, then is 240 furlongs, which is the ſecond terme ſeruing to conclude the Geometricall proportion on this manner.

15 240: *ergo* 9 144

So farre therefore they trauelled, the reſt, to wit, 96 furlongs the enemies went.

A labourer vvas hyred for this vvages, to receiue the firſt day a pennie, the ſecond three, the third fiue, and after the ſame order his wages ſhould be continued for the dayes following: at length being diſmiſſed, he recei-

ued 1500 pence, I demaund how many dayes he ſerued?

Here if you gather together all the termes of the Arithmeticall progreſſion into one ſumme, you ſhall make a way to the Geometricall progreſſion. As for example, if you adde together his wages for 30 dayes, the firſt terme of the progreſſion being an vnitie, and the difference 2, the whole ſumme ſhall be 900 pence: whereupon I may conclude:

If 900 require 30: ergo 1500, 50.

In the examples following, Geometricall proportion muſt goe before to informe the principall proportion.

When a vveb of cloth contayning 40 yardes is bought for 50 crownes, for how much ſhall he ſell the yard, that in an hundred crownes, deſireth to gaine 12?

Before you can conclude the queſtion by Geometricall proportion, being as it were the decider of the whole controuerſie, you muſt ſeeke out the price of one yarde. If 40 yardes are bought for 50 crownes: then one yarde doth coſt 1 $\frac{1}{4}$ of a crowne. This ſhall be the third terme of the principall proportion, in this manner. If of an

100 crownes there ariſe 112: then of

1 $\frac{1}{4}$ there ariſe 1 $\frac{2}{5}$

A Marchant buying at Venice an hundred yardes of ſilke for a crowne a yarde, ſpent

in

in his iourney and for the carriage of them 30 crownes. At Lipſia vvhere the yarde is more by a quarter then the yarde of Venice, he ſolde euery yard for 3 crownes. The queſtion is vvhat he gayned?

In this example addition of whole numbers, together with proportion goeth before. For his ſtocke laide out, and his charges being added, the whole is 130. Beſides, to know how many yardes of Lipſia aunſwere the Venice meaſure, you ſhall worke by proportion on this manner. If $\frac{5}{4}$ of Lipſia meaſure, make a yarde of Venice, then 80 yardes of Lipſia ſhall make an hundred Venice yardes. So that you may readily inferre the principall proportion thus:

If 1 *is ſolde for* 3: *ergo* 80 240

Whereupon 130 crownes being the ſtocke and charges ſubducted out of 240, I ſee he gayned 110.

Another marchant bought three hogsheads of honie. The firſt vvayed 349 pound, the ſecond 286 pound, the third 300, out of euerie hundred are ſubducted tenne pound becauſe of the veſſell, and a pound of honnie is ſolde for 5 pence: I demaund vvhat he is to paie.

For ſo much as the price of one pound of honie is ſet downe, therefore by proportion

ſeeke out the price of an hundred, 1 pound is ſolde for fiue pence: ergo 100, for 500 pence, then adde the waight of the veſſels, the whole is 110, adde alſo the waight of the three hogſheds, the totall is 935: whereupon you ſhall conclude:

110 are ſold at 500 pence, ergo 935 at 4250, that is, 170 florentines.

Another buying two yardes of cloth at three crownes, ſolde ſixe yardes afterwardes for 11 *crownes. Hauing got by this meanes* 46 *crownes, there ariſeth a queſtion howe many yardes he bought, and how many he ſold.*

Before you can come vnto the principall proportion, you muſt order the termes therof by a former proportion, and by ſubduction. Firſt therefore you muſt conſider howe much the ſixe yardes of cloth which afterwardes were ſolde at an higher price coſt at the firſt, the which may eaſily be inferred by that which is preſuppoſed in the queſtion: for if two yardes coſt three crownes, then ſix yardes were ſold at nine crownes. This price I ſubduct from 11, and I ſee that ſixe yardes were ſold for two crownes more then they coſt. So that it is eaſie to reſolue the doubt thus:

If 2 come of 6, then 46, 138

A Vintener bought 124 *hogſheds of vvine on this condition, that ſubducting out of euerie two hogſheds becauſe of the leeſe fiue gallons, he ſhould pay for euery hogſhed* 26 *crovvnes:*

Tell

Tell me vvhat he is to pay.

Proportion going before with multiplication and ſubduction, will ſet in order the principall proportion. Firſt I ſeeke howe much is ſubducted out of al the hogſheds becauſe of the dragges. Out of two hogsheds are drawen 5 gallons, ergo out of 120 hoggesheds are drawen 310. Then I reduce all the foreſaid hogſheds vnto gallons, multiplying them by 32, for ſo many gallons doth euerie hogſhed hold, the *facit* is 3968. Out of theſe I ſubduct according to couenant 310 meaſures: the remainder is 3658 gallons, whereupon the proportion ſhall be thus:

32 *giue* 26: *ergo* 3658 2972 $\frac{1}{8}$

Of two Carpenters, the one alone vvould frame an houſe in 30 *daies, the other in* 40. *Now taking another vnto them, they finiſhed the vvorke in* 15 *dayes. In what time then would the third man haue made it?*

Here muſt you vſe proportion with addition, and ſubduction of fractions. The firſt would haue done the worke in 30 dayes, ergo in 15 daies he finiſhed $\frac{15}{30}$ or $\frac{1}{2}$ of the worke, the ſecond would haue wrought it in 40 daies ergo in 15 dayes he wrought $\frac{15}{40}$ or $\frac{3}{8}$. The partes added together are $\frac{7}{8}$ of the worke: which being ſubducted out of $\frac{8}{8}$, that is, out of the whole worke, there remaineth $\frac{1}{8}$, the third mans worke. In ſo much that the proportion may thus be concluded:

$\frac{1}{8}$ is framed in 15: ergo $\frac{1}{1}$ 120.

Put caſe 14 *pound of ſome vvare were bought for* 6 *crownes, and euery hundred were ſold for* 47 *crownes: ſubduct ſo many pound out of the vvhole as will in the ſale amount to* 80 *crownes gaine. How many pound then vvere there ſold?*

To vnknit this knot you ſhall vſe proportion with ſubduction of mixt numbers. Firſt I ſeeke what he paide for an hundred pound. If 14 pound be bought for 6 crownes, then 100 pound giue $42\frac{6}{7}$. Moreouer for ſo much as an hundred pound were ſolde for 47 crownes, therefore $42\frac{6}{7}$ being ſubducted out of 47, the remainder is $4\frac{1}{7}$ the gaine of an hundred: whereupon I ſay, If $4\frac{1}{7}$ come of 100, then 80 of $1931\frac{1}{29}$.

Sometimes many and ſundrie kindes of numeration, as well out of ſimple as compared Arithmeticke, goe before the principall proportion, wherein the diligent young practitioner may exerciſe himſelfe.

CHAP. XVIII.

Of reciprocall proportion.

HItherto of ſimple direct proportion: now muſt we come to Reciprocall proportion.

Reciprocall proportion is that vvherein the termes being ſet downe croſſewaies, as the third is vnto the ſecond, ſo reciprocally the firſt is to the fourth.

Among

Among the vulgar people we call Reciprocation the going backewarde the ſame way we begunne: which is euidently ſeene in reciprocall proportion being compared with direct proportion.

For that way that direct proportion goeth on forwarde, the ſame way reciprocall proportion returneth backwarde. For in direct proportion the termes are proportionall, not onely as they ſtand in order, but alſo croſſewayes: that is to ſay, as the firſt is to the ſecond, ſo the third is to the fourth, and alſo as the firſt is to the third, ſo the ſeconde is to the fourth. But in reciprocall proportion the termes being taken firſt orderly, are altogether without proportion, and croſſewayes they differ in all reſpects in that one is of greater inequalitie, the other alwayes of leſſe: yea rather as the third is to the ſecond, ſo backwardes, the firſt is to the fourth, in ſo much that that which in direct proportion was the firſt terme, is the thirde in reciprocall proportion: and againe that which is the third in direct proportion, is the firſt in reciprocall proportion.

For example ſake, if twelue hyred ſeruantes doe their worke in 8 dayes, then 24 ſhall doe it in foure dayes. Here are ſet downe foure termes of reciprocall proportion, which being placed directly ſhall ſtande thus:

A	B	C	D
24	8:	12	4

Here as A is to B, ſo is C to D: and croſſewayes, as A is to C, ſo is B to D. But being ſet downe reciprocally, they ſhall be thus placed.

C B A D

Where 12 are not to 8, as 24 vnto 4, for the one is ſeſquialtera, the other is ſextuple, neither as 12 to 24, as 8 to 4, for in the one, the antecedent is ſubdupla to his conſequent, in the other it is double. But as 24 to 8, ſo contrariwiſe, 12 is to 4: that is to ſay, as the antecedent of the latter proportion is vnto the conſequent of the former: ſo reciprocally the antecedent of the former is to the conſequent of the latter. Beholde the platforme and manner of this reciprocation ſet downe in the termes following.

I II III ☞ IIII The places and order of direct proportion.

IIII ☜ III II I The order and places of reciprocall proportion.

But this reciprocation is ſooner perceiued in the thing numbred, then in the bare numbers themſelues, and that by this marke: in that the condition & proprietie of the thing is encreaſed by ſo much the more, by how much the more the price therof, or the waight, or the meaſure, or ſome ſuch like thing is diminiſhed. For generally in it by how much the third terme is greater, by ſo much the fourth is leſſer, and contrariwiſe.

Recipro-

Reciprocall proportion hath this proprietie: the facit of the two former termes is equall to the facit of the latter.

From hence is there drawen a Rule, which when there are giuen three numbers reciprocally proportionall, diuiding the *facit* made of the first and second, by the third findeth out the fourth proportionall number.

As in the former example, 12, 8, 24, 4, the *facit* 96 made of 12 and 8 the two former numbers, is equall to the *facit* made of 24 and 4, so that the manner of working may well be concluded: if you diuide the *facit* 96 made of the first and second numbers by 24, the third in order for the quotient will be 4, which is the fourth proportionall number.

At Basill the Bakers lawe commaundeth that when 8 bushels of corne are solde for 40 pence, the halfpeny-manchet shall waigh seuen halfe ounces, what shall the loafe waigh when wheat is sold for 70?

I call here an halfepenie the $\frac{1}{12}$ of a pennie at Basill, whereof 25 make a crowne. In this example reason affordeth me this, that the price of a loafe continuing at one stay by how much the more the corne waxeth dearer, by so much the more the waight of the loafe must be lesser, & contrariwise by how much the more the dearth decreaseth, by so much the more the waight of the loafe must encrease: whereby I perceiue, that in this que-

ſtion I muſt vſe the rule of reciprocall proportion on this manner:

If 40 *require* 7: *then* 70 *how many?*

Multiply 40 by 7, the *facit* ſhall be 280, which being diuided by 70, giueth 4 or two ounces, which is the waight of the loafe.

Three milles in two dayes grinde 20 *buſhels of corne, in what time then ſhall fiue milles grind ſo many buſhels?*

Here againe by how many more milles there be, in ſo much leſſe time they ſhal grind the corne. Wherefore the proportion ſhall be concluded reciprocally in this manner.

3 *grinde* 2: *ergo* 5 1 $\frac{1}{5}$.

A certaine man borrowing 66 *crownes of his friend, and repaying them at ſeuen monethes end, promiſeth to pleaſure his creditor in as great a matter. Whereupon the creditor afterward deſired to borrow of him* 112 *crowns, but they doubt how long time he ſhould keepe the money, that the lending of his money might be correſpondent.*

For ſo much as the latter man deſired to borrow the greater ſumme, therefore he is bound to reſtore it ſooner then the firſt ſhold, whoſe ſumme was leſſe: whereupon the reciprocation ſhall be concluded thus:

66 *giue* 7: *ergo* 112 4 $\frac{1}{8}$.

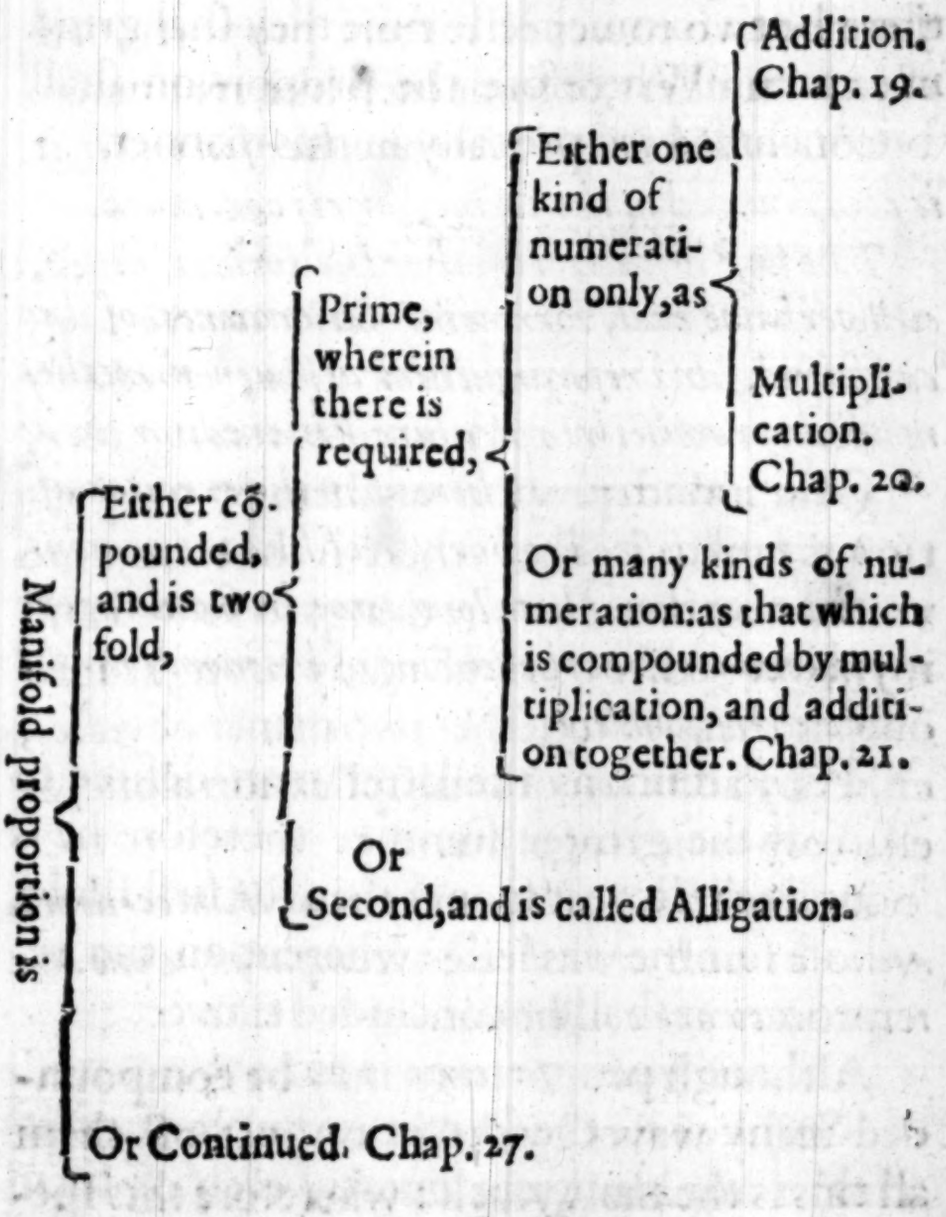
So likewiſe the former example of reducing 126 Hungarie ducats (each of them being

being valued at 28 vrſati) vnto Thaleri being worth 18 vrſati, may be handled and reſolued after this faſhion.

28 126: 18 196.

CHAP. XIX.

Of manifold proportion, and firſt of that which is compound by Addition commonly called the rule of Fellowſhippe.



Hitherto hath bene handled ſimple proportion, as well direct as reciprocall, being ſet forth by diuerſe examples: now followeth manifold proportion.

Manifold proportion is that vvhich conſiſteth of more then foure termes.

Simple proportion diſiunct, conſiſteth of foure termes, but manifold proportion conſiſteth of more, as when in one and the ſame proportion there be manie conſequents to one antecedent, or contrariwiſe, or elſe when there be two rowes of nũbers proportionall in two numbers, or ſome ſuch like manner.

Manifolde proportion is double: for either it is compounded in the termes, or els continued.

The firſt ioyneth together the termes giuen, either by one onely kinde of numeration, to vvit, either by addition or multiplication, or elſe by more, as by addition and multiplication together.

Compound proportionalitie or proportion is ſaid to be that wherein the termes are vnited together. But the things ſo vnited and ioyned together, are of many brought into one, and heaped together two maner of waies, either by addition or multiplication alone, or elſe both waies together.

By Addition, as when many as well antecedents as conſequents are taken and compared, as one antecedent to one conſequent.

Although proportions may be compounded many wayes by Addition, yet of them all this is the moſt vſuall: wherefore this ſpeciall

ciall definition ſeruing moſt for this purpoſe is taken in ſtead of that which is more generall. For example ſake, if 3 vnto 2, be as 9 vnto 6, then by addition 3 and 9 that is 12, ſhall be in like manner vnto 2 and 6, that is 8, ſo that the termes of the proportion may ſtand as you ſee,

12 vnto 8 : are as 3 to 2 and 9 to 6

In this kind of manifold proportion, the rule following is to be vſed,

If in the termes of manifold proportion the conſequent of the one proportionality be giuen added together, and the antecedents ſeuerally by themſelues, then as the antecedents knit together by addition, are vnto the conſequents added, ſo ſhall the ſeuerall antecedents be vnto their ſeuerall conſequents, and ſo contrariwiſe.

This is the 12 p. 7, ſauing that it is reuerſed and applyed ſomewhat more fitly to this purpoſe. For the words of the propoſition be theſe: *If there be a multitude of numbers, how many ſo euer proportionall, as one of the antecedents is to one of the conſequents, ſo are all the antecedents vnto all the conſequents.* And the ſame backeward is true alſo. By which conuerſion it appeareth how the examples which commonly are referred vnto the rule of fellowſhippe, hauing many proportions coupled by addition, are to be handled and to be

reſolued by the rule of proportion, in which examples the antecedents of the one proportion are ſet downe ſeuerally, to wit, the contributions of the partners, and the conſequents are added together, to wit, the common gaine or loſſe. So that the contributions and common ſtocke of the partners, or elſe generally all thoſe numbers into which the other is to bee diſtributed proportionally muſt be added together, and the totall muſt be ſet in the firſt place in ſtead of the antecedent of the firſt proportion, whoſe conſequent muſt be the foreſaide number which is taken to be diſtributed proportionally. In the third place the termes compounded by addition muſt be placed ſeuerally, & the rule muſt be repeated as often as there be ſeuerall numbers, whereupon I may conclude the proportion. As in the examples following.

Three men being partners laide their money together, the firſt 60, *the ſecond* 100, *the third* 135 *crownes. Nowe they gayned after a vvhile* 45 *crownes. The queſtion is howe much of the gaines each of them by duetie ought to haue.*

In this and ſuch like examples equitie and reaſon requireth, that that which is gotten by the common ſtocke ſhould alſo be common, and ſhould be ſo diſtributed, that euery mans portion ſhould haue the ſame proportion to the whole gaine, that the mony which he

he laide downe had vnto the whole ſtocke: and croſſewayes as the whole ſtocke is vnto the whole gaine, ſo then euery mans portion laide together, ſhould be vnto his part of the gaines. Wherefore here this queſtion of manifold proportion is manifeſt. Nowe the termes giuen are theſe:

The firſt propor.		The ſecond propor.	
I	II	III	IIII
		60	
	45	100	
		135	

The conſequents of the ſecond proportion are ſet down but ioyntly together, to wit, 45, the antecedents are ſeuerall. Wherefore to conclude the ſeuerall conſequents, I ſet the whole ſtocke gathered by adding of 60, 100, 135, to wit, 295 in the firſt place, which is vnto the whole gaine, as the ſeuerally termes in the third place are vnto the ſeuerall termes in the fourth place, which we deſire to know. The example is thus:

			60	9	$\frac{9}{59}$
295	45	: ergo	100	15	$\frac{15}{59}$
In the leaſt termes 59	9		135	20	$\frac{55}{59}$

Three partners in a common ſtocke of 500 *crownes, loſt* 124, *to the repayring vvhereof the firſt vvas bound to be contributarie* 20 *crownes, the ſecond* 43: *I demaunde howe much euery one put into the common purſe.*

Here wee haue of the latter proportion three antecedents, two are aſſigned in expreſſe words, but the third is ſet downe ſomwhat obſcurely. But for ſo much as the totall of the three added together is ſet downe, to wit, 124, the third may eaſily be knowen by ſubduction. Adde therefore 20 and 43, the totall is 63, which being ſubducted out of 124 there remaines 61, which is the third mans portion in the loſſe. Wherefore as in the former example we gathered euery mans gaine by euery ones contribution, ſo likewiſe in this place by the partes of the loſſe we may gather each mans ſtocke. The example is thus:

			20	80	$\frac{20}{31}$
124	500:	ergo	43	173	$\frac{12}{31}$
Or 31	125		61	245	$\frac{30}{31}$

Cicero in his Oration for Cecinna ſayeth: A woman making her vvill departed this life, ſhe made Cecinna heire of eleuen ounces and an halfe of her goods, and Marcus Fulcinus the free man of her former husbande heire of two ſixt parts of an ounce. Vnto Eutius ſhe gaue one ſixt part. How much then had euerie one for his portion.

In this example the whole patrimonie is diuided into eleuen ounces, halfe an ounce, and three ſixt partes of an ounce: the which parts are the antecedents of the late proportion. But in that they be of diuerſe denominations

nations, I resolue them all into the least denomination, to wit, into sixts. If then you allow to an ounce sixe sixths, and to the pound twelue ounces, there shall be in a pound 72 sixths. Whereof 11 ounces and an halfe are worth 69, and two sixths 2, one 1. Whereupon supposing the whole patrimonie to be 2500 crownes, the termes shall be thus:

			69	2395	$\frac{5}{6}$
72	2500 :	ergo	2	69	$\frac{4}{9}$
			1	34	$\frac{13}{18}$

Three couenanted betweene themselues, that of their gaines the first should haue twice as much as the second. Now they had laid downe 24 crownes, and the third 36. But when they had gotten 12 crownes they cannot tell howe to diuide them lawfully according to their couenant.

Here that which the first man laide down being couertly expressed, must be gathered out of the couenant. For in that he was to receiue twice so much as the second, it is manifest that that which he laide out was twice as much as that whereof the second was contributary, whether it were money, or whether it were his labour, or whether it were valued in both. Wherefore if for the first you take 48 crownes the double of 24, then they being added together, the whole doubt shall be remooued by the rule on this manner.

			48	$5\frac{1}{3}$
108	12	ergo	24	$2\frac{2}{3}$
9	1		36	4

The good man of an houſe bequeatheth vnto foure men 584 *crownes, to be diuided after this rate, that A ſhould receiue* $\frac{1}{2}$, *B* $\frac{2}{3}$, *C* $\frac{3}{4}$, *D* $\frac{5}{6}$, *vvhat vvas euery ones portion?*

Here the proportion of the parts vnto the goods which were left is expreſſed: yet the parts themſelues in the goods doe not yet appeare manifeſt. The leaſt number therefore that may be diuided by the denominators of the fractions, which is 12, will declare the proportion. For $\frac{1}{2}$ of 12, are 6, $\frac{2}{3}$ are 8, $\frac{3}{4}$ are 9, $\frac{5}{6}$ are 10. Theſe being added, the example ſhall be thus:

			6	$106\frac{2}{11}$
33	584	ergo	8	$141\frac{19}{33}$
			9	$159\frac{3}{11}$
			10	$176\frac{22}{33}$

But if he had vvilled to make ſuch a diuiſion among them that were to receiue the legacie that the firſt ſhould haue $\frac{1}{2}$ *and* $\frac{1}{5}$ *wanting* 20 *crownes, the ſecond* $\frac{1}{3}$ *and* 30 *crownes, the third* $\frac{3}{4}$ *and* 10 *crownes, the fourth* $\frac{1}{6}$ *wanting* 12 *crownes: then vvhat ſhould their portions be?*

In this caſe that which is more then their portions I take it from the whole ſubſtance, that which wanteth I adde vnto it: as 30 and 50 are 80, which being taken from 584, there remaines

remaines 504. Hereunto adde 20 and 12, or 32, the totall is 536. Then I ſeeke out the leaſt number that may be commonly diuided by the denominators, to wit, 60, the partes of the ſame denomination are for the firſt 30 and 12, for the ſecond 20, for the third 45, for the fourth 10: and then the portion of the firſt being firſt added together, and afterward all the reſt, the example will be thus:

			40	192 $\frac{48}{117}$
117	536	ergo	20	91 $\frac{73}{117}$
			45	206 $\frac{18}{117}$
			10	45 $\frac{95}{117}$

Then that the teſtator may be ſatisfied, from the firſt mans portion I take 20 crowns there remaines 172 $\frac{48}{117}$, vnto the ſecond adde 30, they make 121 $\frac{73}{117}$, vnto the third adde 50, his whole portion will be 256 $\frac{18}{117}$, laſtly from the fourth take 12, there remaine vnto him 33 $\frac{95}{117}$, all which partes are anſwerable vnto the 584 crownes, which were to be diuided.

Thre partners laide together three equall portions, but they left them there, ſome for longer time and ſome for ſhorter: the firſt left his money in the ſtocke two monethes and an halfe, the ſeconde three monethes and $\frac{2}{5}$, the third foure monethes and $\frac{2}{3}$. Put caſe they gained 12 crownes, I demaund what was euery mans portion.

This example being of mixt numeration, requireth no other worke then the former. For the ſeuerall times being added together, wherein they left their ſtocke for the common trafficke, the totall will be $\frac{817}{30}$: So that the example will be.

$\frac{817}{30}$ 12

$\frac{5}{2}$ 2 $\frac{266}{317}$

$\frac{17}{5}$ 3 $\frac{273}{317}$

$\frac{14}{3}$ 5 $\frac{95}{317}$

CHAP. XX.

Of multiplied proportion compound by multiplication, vvhich commonly is called the double Rule.

THe former examples appertained vnto proportion compound by addition: now followeth that vvhich is compound by multiplication, vvhich in ſtead of two termes of the proportion, vſeth the facit *made of them.*

This is the right compounding of proportion, which gathereth them together by multiplication as was ſaid before, and appeareth by the fift definition of the ſixth booke of Euclide, becauſe it doth not only ioyne together the termes themſelues, but alſo maketh the proportion of the compound termes to comprehend the proportion of the ſimple. Wherefore hitherto are to be referred, and

by

by this rule of proportion are to be handled theſe examples, wherein two termes by multiplication muſt be reduced vnto one: for there proportion as appeareth by the fifth prop. of the eighth booke of Euclide, is gathered of the proportion of the termes multiplied one by the other. Of this ſort are all thoſe examples which commonly are referred to the double rule, wherein two termes are ſet in the ſame places, that is, the principall proportion hath a certaine circumſtance adioyned vnto it, in ſo much that here the proportion ſhould be twice concluded, were it not that for thoſe two termes the products made of them are taken as one.

The termes propounded by multiplication, are either of the ſame place, as in direct iproportion, or of diuerſe places, as in reciprocall proportion.

The examples of the firſt are theſe.

Foure Students ſpend in 3 monethes 19 crownes: how much therefore ſhall eight Students ſpend in 9 monethes.

You ſee here the principall termes haue annexed vnto them a circumſtance of time, & therfore (the maner of ordering the termes in the golden rule being obſerued) the firſt and the third are double, on this faſhion:

4 19 8
3 9

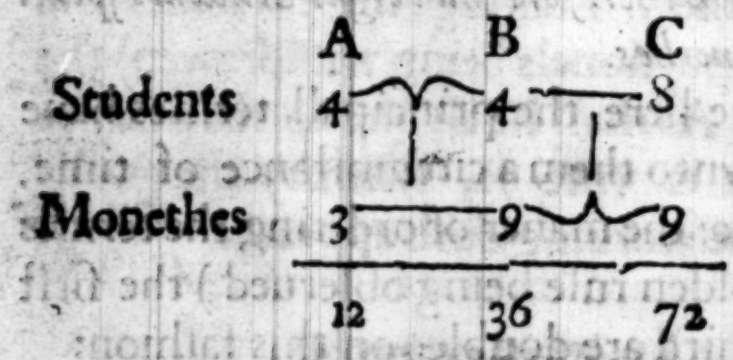
Wherefore for thoſe two I take the compounds by multiplication. As for 4 and 3

I take 12, for 8 and 9, 72, to wit their products: So that the compound multiplication ſhall be concluded thus:

12 19 72 114

Or 1 6

Here betwene the expences of foure ſtudents in three monethes, there is obſerued the ſame proportion which is betweene the expences of 8 ſtudents in 9 moneths, to wit, the proportion compound of the two, to wit, of 4 vnto 8, and 3 vnto 9, or to ſpeake more plainely, the expences are one to another, as thrice foure vnto nine times eight. The which may eaſily be proued, if you put in the meane proportionall number to whom the termes of the antecedents and conſequents may belong. So that if you ſhould ſet it downe that foure ſtudents in 9 monethes ſhould make their expence, the example will be thus:



Here then if foure ſtudents in three monethes diſpende euery one but one, then ſo many ſtudents in nine monethes ſhall ſpende thrice as much. Wherefore the proportion

of

of the expences of A vnto B, ſhall be the ſame that is betwene three and nine. Likewiſe if foure ſtudents in nine moneths ſpend euerie one but one, then 8 ſtudents in as manie monethes will ſpende twiſe as much. Whereupon there will be the ſame reaſon betweene B and C, as is betweene 4 and 8.

Furthermore if you multiply 3 and 9, the termes of the firſt reaſon by one and the ſame number, namely by 4, the productes, namely 12 and 36, ſhall keepe the ſame reaſon by the 17 propoſition of the 7 of Euclide: ſo ſhall 36 and 72 being made of 4 and 8 multiplyed by 9. Whereby it is gathered that as the charges of A is vnto B, ſo is 12 vnto 36. And in like manner as B is vnto C, ſo is 36 vnto 72. To conclude, ſeeing there are here three proportionall numbers, A, B, C, and as many more in number anſwerable vnto them 12, 36, 72, therefore by the 14 of the 7 (taking the extremes the meane proportionals being withdrawen) the charges of A ſhall be all one to the charges of C, as 12 is vnto 72, whereby the goodneſſe of the working doth appeare.

An other example.

Two Printers in foure dayes print 16 formes, how many ſhall ſeuen Printers print in 14 dayes?

Here the placing of the termes prescribed in the rule of proportion being obserued, they shall be set in this manner:

Printers.	2	16	7
Dayes	4		14

Then, the first termes and the last being multiplied one by another, the proportion is concluded thus:

8 16: therefore 98 196

Eight yardes of cloth, 4 yardes and $\frac{1}{4}$ broad are bought for 11 *crownes: therefore how shall* 15 *yardes of cloth be bought* 1 *yarde* $\frac{3}{4}$ *broad?*

This example of mixt numeration varieth nothing from the former, therefore it is thus dispatched:

8	11	15	
$1\frac{1}{4}$		$1\frac{3}{4}$	
$\frac{40}{4}$	11: therefore	$\frac{105}{4}$	$28\frac{7}{8}$

Three men trading together by ill lucke lost 52 *crownes. The first put in* 110 *crownes for* 5 *monethes, the second put in* 84 *crownes, the third* 65, *I know not for what time. Their traficke being ended, the first found that he had lost* 22 *crownes, the second found he had lost* 18, *the third* 12. *How long therefore was the money of the last two in the common stocke?*

I thinke that this example (though it seeme to belong to the Rule of Fellowship) may well be referred to the rule of proportion

tion compounded by multiplication, yet it is vnlike to the other, becauſe of the numbers giuen, the middlemoſt onely is compounded by multiplication, the other are ſought out by the rule of three, proceeding from a terme known to thoſe which are vnknowne. Therefore the firſt mans 22 crownes being multiplied by his 5 moneths, the *facit* is 550. Whereupon I conclude, if his money, that loſt 22 crownes, multiplied by the time doth make 550, what ſhall the loſſe of 18 crownes? and what ſhall the loſſe of 12 crownes make? The termes ſhall ſtand thus:

			18	450
22	550:	therefore		
			12	300

The fourth numbers inferred by the rule of three are compounded of ech mans ſtocke multiplyed by the time for which it was laid out. Therefore ech of them ſeuerally diuided by his owne ſtocke, the time is ſeuerally inferred, ſo that the ſecond mans money was out of his handes fiue monethes and $\frac{5}{14}$, the third mans money was out foure and $\frac{8}{13}$ of a moneth.

An abridgement of the former worke.

If the termes in the firſt and third place fall out to be equall, then they being taken away, the reſt of the termes ſhall inferre the proportion.

The reaſon of this abridgement ariſeth

out of the 17 propoſition of the 7, for the numbers giuen keepe the ſame proportionalitie which they would haue being multiplied by one and the ſame number. As:

If the gaine of 25 *crownes in foure yeare be eight crownes, vvhat ſhall* 100 *crownes yeeld in foure yeare?*

Here the termes are giuen as you ſee.

Stocke	25	8	100
Yeares	4		4

Forſomuch therefore as 25 hath the ſame reaſon to 100 which they would haue being multiplyed by 4, as it appeareth by the 17 prop. 7, therefore the equall numbers which are in the firſt and thirde place, being caſt away, I conclude the proportion in ſimple termes thus:

25 8: therefore 100 32

Likewiſe the queſtion being thus propounded, the gaine of 25 crownes in 4 yeare, is 8 crownes, therefore what ſhall 100 yeeld in 25 yeares? The termes ſhall ſtand thus:

25	8	100
4		25

Here againe the alternate termes of the firſt and third place being omitted, the reſt ſhall conclude the proportion thus:

4 8: therefore 100 200

If the proportion chance to be reciprocall, then are the termes of the firſt and third place compounded by alternate or croſſe multiplication, and

the queſtion afterward is concluded directly.

Some tranſpoſe the principall termes ſetting the firſt in the third and contrariwiſe the third in the firſt place, & then bring the twofold termes into one by multiplication. The example:

Eight horſes in 12 *daies eate* 9 *buſhels of oats, in how many dayes ſhall* 18 *horſe eate* 24 *buſhels?*

I eaſily perceiue that there is here a reciprocation, for 18 horſes will ſpend in a great deale leſſer time that heape of oates, which 8 horſe will conſume in 12 daies. Therefore by how much the third terme is the greater, by ſo much the fourth terme is the leſſer. Therefore I muſt take heede that diſpoſing the termes as I did before, I conclude not directly the reciprocall proportion by multiplying of the firſt and laſt termes one by another. The termes in the example are thus:

Horſes 8 12 18

Buſhels 9 24

Here therefore I multiply 9 by 18 and make 162 the firſt terme, likewiſe I multiply 8 by 24 and make 192 the third terme, whereby the lawfull induction of the proportion is thus diſpatched:

162 12 : therefore 192 14 $\frac{2}{9}$

If 100 *crownes doe giue* 5 *crownes intereſt euerie yeare, in vvhat time vvill* 56 *crownes giue* 12?

In this queſtion for ſo much as a greater intereſt is ſought for by a leſſer ſtocke of money, the reciprocation is manifeſt, the which intereſt craueth ſo much the more time by how much the more the ſtocke is leſſer. Therefore the termes being ſet downe as it is conuenient, and multiplied croſſewayes, the doubt is anſwered by the rule of three.

CHAP. XXI.

Of manifold proportion compounded by multiplication and addition.

THus much concerning the examples of proportion compounded by addition or multiplication onely: now muſt vve proceede to that proportion vvhich ioyneth both kindes of numeration together.

Therefore proportion compounded by multiplication and addition, is that vvhich firſt of all multiplieth the manifold termes giuen one with another, and then addeth their productes together.

This kinde of proportion by a two folde, yea ſometimes by a fourefolde compoſition, bringeth many termes into one. Vnto this

apper-

appertaine the examples of the Rule of ſecond fellowſhip, as they commonly cal them, wherein the time is annexed to the ſtocke, or ſome other circumſtance to the principall terme. The Theoreme following ſerueth for the working of thoſe examples.

If among the termes of the manifold proportion, the conſequents of one reaſon be giuen added together, but ſome antecedents be giuen ſeuerally and double: then as the antecedents compounded by multiplication and addition are vnto the conſequents added together, ſo ſhall the ſeuerall antecedents compounded by multiplication be vnto the ſeuerall conſequents.

As thus for example.

Two men in common trafficke got 60 crownes, the firſt brought 50 crownes for two yeare, the ſecond 15 for fiue yeare: vvhat ſhall be ech man his ſhare.

In this queſtion containing a partnerſhip of equall contributions in regard of the diuerſe times, I ſee the diuiſion of the gaine muſt be made according to the proportion of the time. That this diuiſion may be made the conſequents of the latter reaſon are giuen added together, namely 60, but the antecedents are giuen not only ſeuerally, but alſo double, as you ſee here.

The first reason.		The second reason.	
I	II	III	IIII
		50	2
60			
		15	5

Therefore first of all I compound the double termes by multiplication, so that after a sort I reduce the vnequall times to an equalitie, the products are 100 and 75, which are all one as if you should say, that the first man should haue as much gaine by 100 crownes in one yeare, as he should haue for 50 crownes in 2 yeare. In like manner the second man should get by 75 crownes in one yeare, as much as he should get by 15 crownes in fiue yeare. Moreouer these two products 100 and 75 set downe seuerally, shall possesse the third place of the proportion: but added together they shall stand in the first place: nowe the rule of three twice repeated, shall deliuer the numbers sought for in this manner:

		100	$34\frac{2}{7}$
175	60: therefore		
		75	$25\frac{5}{7}$

I doe of set purpose omit the demonstration of this composition of the termes, because it is not much vnlike the former.

A hundred and sixtie footmen and fortie horsemen got a bootie of 138 *crownes to be diuided betweene them so, that as often as the footmen receiued one, the horsemen shoulde receiue*

three.

three. How much were the footemen, howe much were the horſemen to haue?

Here likewiſe I place the conſequents of the ſecond reaſon added together in the ſecond place, namely, 138: in the thirde place I ſet the products made by multiplying the numbers ſignifying how often ech one ſhold take his ſhare by the number of the ſouldiers: namely, 160 and 120, theſe added together and making 280, ſhall haue the firſt place. The example is thus:

160 78 $\frac{6}{7}$

280 138: therefore

120 59 $\frac{1}{7}$

Three Butchers hyred a meddow together, promiſing to pay yearely rent for it 30 crownes, the firſt fedde in it 20 Oxen 70 daies, the ſecond fedde 46 Oxen 56 dayes, the third fedde 32 Oxen 60 dayes: How much of the rent ſhall each partner pay?

Againe therefore I multiply each mans heard by the ſeueral times, the products 1400, 2576, 1920, ſhall poſſeſſe the third place: and being added together, they ſhall haue the firſt place, the yearely rent ſhall be in the middeſt thus:

1400 7 $\frac{91}{737}$

5896 30: therefore 2576 13 $\frac{79}{737}$

1920 9 $\frac{567}{737}$

Three marchants were partners for a yeare. The firſt in the beginning brought 250 crownes, but after 3 moneths he withdrew 100. The

ſecond after 2 moneths brought 180 crownes, but after 6 moneths of their partnerſhip were expired, he tooke away 50. The third after three moneths brought in 235 crownes, and after fiue moneths added 45 crownes. Now hauing gotten 68 crownes, what is each mans ſhare?

Theſe kind of examples being in outward ſhew moſt intricate, require a litle more labor, otherwiſe they are to be handled by the ſame art. They wil ſoone be diſpatched, if you indeuor but to ſeuer euerie mans time accordingly as he changed his ſtocke, and make as many multiplications as there were changes. As in this preſent example: Becauſe the firſt man left in the partnerſhip 250 crownes for three moneths, let them be multiplied together, the product will be 750. Then withdrawing 100 crownes, he left but 150 for the nine moneths remaining. Wherfore theſe being againe multiplied by their time, make 1350. Which numbers being added, the totall ſumme for the firſt partner is 2100.

Likewiſe the ſecond man his 180 crownes being multiplied by the 4 moneths wherein he left them in the common trafficke, doe make 720. Moreouer 50 crownes being ſubducted from 180 (for ſo manie he fetcht away from the principall after the ſixth moneth of their partnerſhip) there remaine 130 crownes, which if you multiply by the ſixe moneths remaining, the product will be 780,

which

which added to the former, maketh 1500.

So also multiplie the third man his 235 crownes by 9, they make 2115. Item 45 by 7, they make 315, which added to the former, make 2430. These termes being thus compounded, if you proceed as you did before, the proportion will be inferred, which here you see set downe, with the ordering of the whole example.

	The stock.	*Moneth.*	*Numbers cõpounded first by multiplicatiõ.*	*Numbers cõpounded by addition.*	
	250.	3.	750.		
I.				2100.	
	150.	9.	1350.		
	180.	4.	720.		
II.				1500.	
	130.	6.	780.		The totall summe.
	235.	9.	2115.		6030.
III.				2430.	
	45.	7.	315.		
			2100.	$23\frac{137}{201}$	
	6030.	68.	1500.	$16\frac{184}{201}$	
			2430.	$27\frac{81}{201}$	

CHAP. XXII.

A treatise of Alligation, whereof B. SALIGNACVS was the Author.

- In Alligation we are to consider,
  - The proprietie which is this: that in alligation,
    - The measures should be like.
    - The price of the mixt measure should be meane in quantitie betweene the prices of the simple measures.
  - The kinds which are two
    - Prime: which coũterchaungeth the differẽces betweene the extremes & the mean: & herein we are to consider the
      - Chap. 23. Propriety, which is this, that if the extremes are in number
        - Euen: ech of them seuerally must be cõpared once only with the meane.
        - Odde: then
          - Euery extreame of the greatest number must be compared with the meane once onely.
          - Euery extreame of the least number must be cõpared diuerse times with the meane.
      - Kindes, which are two,
        - The first kind, some part of the nũbers sought for being giuen, inferreth the rest by simple proportion. Chap. 24.
        - The second kind, the totall summe made of the numbers sought for being giuen, inferreth all the particulars by proportiõ compoũded by addition. Chap. 25.
    - Second: which inferreth the meane by proportion compounded by multiplication and addition. Chap. 26.

Of

Of the Definition and proprieties of Alligation. Chap. 1.

HItherto we haue ſpoken of the firſt kinde of compound proportion: the ſecond followeth, and is called Alligation.

Alligation is an art, vvhich (by the meanes of certaine things giuen) maketh equall the totall made of the prices of the meaſures mingled together, vnto the totall made of prices of the meaſures taken ſeuerally.

There is 100 pound waight of ſiluer worth 17 pound to be mingled with other ſiluer worth 24 pound, ſo that the totall made of the prices of the waightes mingled together may be equall to the totall made of the prices of the waights taken ſeuerally. Here all the prices are giuen, the art therefore which by the meanes of theſe prices giuen maketh equall the totall numbers is Alligation.

Item, Of two kinds of corne mingled together ſuppoſe there were 10 buſhels worth 16 ſhillings, and 18 buſhels worth 12 ſhillings: foure buſhels of this mingled corne are to be ſold, ſo that the totall ſumme made of the prices of the 28 buſhels mingled together be equall to the totall made of the prices of the 10 and 18 buſhels ſeuerally taken. Here both the number and the price of the ſimple meaſures are giuē, but there is but only a certain part giuen of the number of the meaſures mingled together. The art therefore which by the meanes of the thinges giuen maketh

equall the totals propounded in the queſtiõ, is called Alligation.

The propertie of Alligation is this, that the meaſures in it be alike: but the price of the mingled meaſure muſt be in quantitie meane between the prices of the meaſures ſeuerally giuen.

A buſhell of corne worth 16 ſhillings is ſuppoſed to bee mingled with a buſhell of corne worth 18 ſhillings. Here by this property firſt of all the buſhels are like meaſures. Then the price of a mingled buſhell muſt be in a quantitie meane betweene 16 and 18. that is to ſay, it muſt be greater then 16, and leſſe then 18. So that the price of a ſimple meaſure is called the Extreme, but the price of a mingled meaſure is the Meane.

CHAP. XXIII.

Of Prime Alligation, and the propertie thereof.

ALLigation is two fold. Prime, which doeth counterchange the differences of the termes from the meane.

By the definition of Alligation generally taken it appeareth, that in all alligation there are certaine things giuen. Therfore in Prime alligation generally let all prices be vnderſtood to be giuen. Moreouer to coũterchange the differences of the extremes, is nothing elſe but to attribute the difference of the leſſer extreme to the greater extreme, and contrariwiſe

wise to attribute the difference of the greater extreame to the lesser extreame. Nowe this counterchaunge is therfore vsed, that we may thereby make equall the totall made of the prices of the measures seuerally taken vnto the totall of the prices of the measures mingled together. That this may be the better demonstrated, we must first of all set downe two premisses. The first is this:

If one and the same number doe multiply certaine numbers seuerally, and the totall made of them being added together, the product made of the totall shall be equall to the totall made of the products of the parts.

As here: let 10 be the totall number made of 4 and 6, let the number multiplying them all, be 2, the products made of 4 and 6, are 8 and 12, and the totall made of them is 20. Therefore the product made of 10 by 2 shall be 20.

The second premisse is this:

If three vnequall numbers being giuen, you multiply any one of them by the other two remayning, and augment the selfe same number by one of them which remaine, and diminish it by the other: and then multiply the number augmented by that which was taken away, and the number diminished by that which was added to: the totall number made of the latter products shall be equall to the totall made of the former products.

As here: Let there be three vnequall num-

bers 4, 7, 9, multiply 9 by 4 and 7, and let the products be 36 and 63, then augment and diminish the selfe same number 9 as is aforesaid, adde therefore 7 vnto it, and let the totall be 16, also from 9 take 4, and let the remainder be 5, then multiply 16 by 4, and 5 by 7, and let the products be 64 and 35: here the totall made of 64 and 35, shall be equall to the totall made of 36 and 63.

These things being thus set downe: nowe of two kindes of wine, let one be worth 14 pence, another worth 11 pence, let the difference be 2 and 1 from the meane, which is 12 pence. The totall made of the differences is 3. The meane price is 12, the extreames are 14 and 11. Therefore multiply the meane which is 12, by the totall made of the differences, namely by 3, the product is 36, that is to say, three pottles worth 12 pence a peece shall be worth 36 pence, I say therefore that one pottle worth 14 pence, and two pottles worth 11 pence a peece shall be worth 36 pence. That is: I say that the totall made of 1 and 2, by 14 and 11, is 36. For here 3 is the totall made of 1 and 2 as was aforesaid. Therfore by the first premisse if you multiply 12 by 2 and 1, the totall made of the productes 24 and 12 shall be equall to the product made of 12 and 3. But 12 multiplied by 3 maketh 36, therefore the totall of the products made of 12 multiplyed by 2 and 1 shall be 36.

Againe these three numbers 11, 12, and 14

are

are vnequall, the differences of the extremes 14 and 11 from 12, which is the meane, are 2 and 1. The greater extreame 14 is 12 more by 2, the lesser extreme 11 is 12 lesse by 1. Therefore by the second premisse if you multiply 12, augmented by 1, which was taken from it, and 12 diminished by 2, which was added to it: that is to say, if you multiply 14 by 1, and 11 by 2, the totall made of their products shall be equall to the totall made of the products of 12 multiplyed by 1 & 2. But the total of the products made of 12 by 1 and 2 is 36, as it appeared before: therefore the totall of the products made of 1 and 2 by 14 and 11, shall be also 36, which was the thing to be demonstrated. Therfore the reason why with vse this counterchaunging is as you see.

Here this also is to be noted:

If ech number of the extreames be a number of multitude, the differences may be counterchaunged diuersely: and therefore then in one and the same example a diuerse alligation may be made.

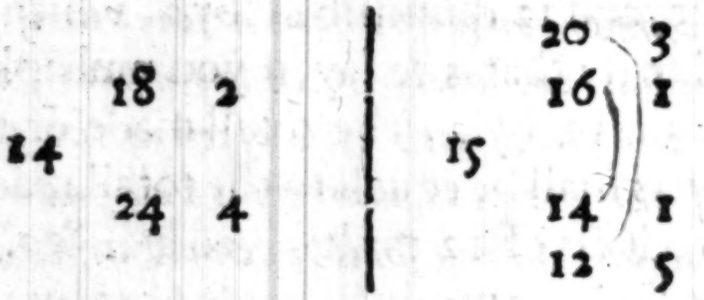
But all Alligation maketh equall the totall made of the prices of the mixt measures to the totall made of the measures seuerally taken, as appeareth by the definition of generall alligation: wherefore in the alligation intended when each number of the extremes is a number of multitude, the counterchanging of the differences is at our owne choise.

Hereafter followe two proprieties, and as

many kindes of prime alligation. The first proprietie is thus:

If the extremes be equall in number, each of them is compared with the meane but once onely.

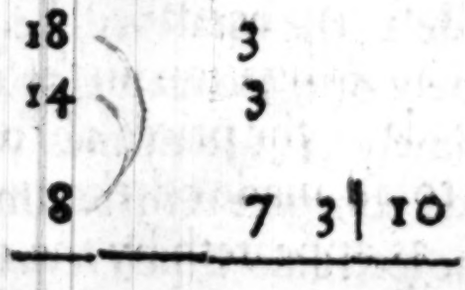
As here:



The second proprietie is thus:

If the extremes be vnequall in number, then each of the extremes whose number is the greater, are compared vvith the meane once onely: but touching the lesser number of extremes, if the extreme be but one onely, then is that one extreme to be compared vvith the meane so often as there is vnities in the greater number of the extremes.

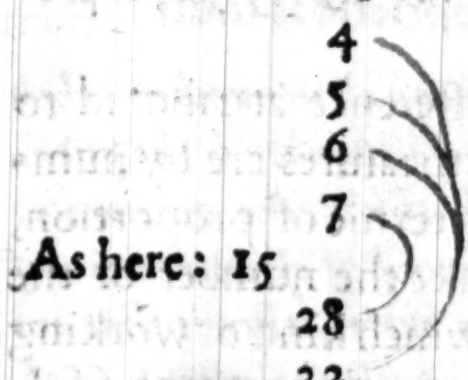
As here:



In this example the greater number of the extremes is 2, and therefore the extreme of the lesser number is twise compared with the meane. Where also you shall note that the totall made of the manifold differences attributed

buted to one and the ſame extreme, is taken for one difference : for, for 7 and 3 wee take 10.

But if the extremes of the leſſer number be many, if you compare more then one of them oftentimes vvith the meane, the number ariſing of the compariſon is vvithout arte: but if you compare but one onely vvith the meane, that ſhall be compared with it ſo often as the difference of the vnequall numbers is being added to an vnitie, but the reſt ſhall be compared with the meane but once onely.



In this example the greater number of the extremes is 4, the leſſer is 2, the difference of theſe vnequall numbers is 2, vnto which if you adde 1, the totall will be 3. Now I compare onely one extreame of the leſſer number, namely 32, oftentimes with the meane 15, and I compare it three times, but I compare the extreme which remaineth but once onely with the meane. And then laſt of all, I take the totall made of the manifold differences attributed to one extreme, for one difference as I did before.

Thus much concerning the two proprie-

ties of prime alligation: now follow the two kindes for the vnderſtanding whereof we are to note that the numbers ſought for are the numbers of the ſimple meaſures.

CHAP. XXIIII.

Concerning the firſt kinde of prime Alligation.

PRime alligation of the firſt kind, is vvhen ſome part of the numbers ſought for being giuen, vve inferre the reſt by ſimple proportion.

Here we take the difference attributed to that extreme, whoſe meaſures are by number giuen, for the firſt terme of proportion, and for the third we take the number of the meaſures giuen. The which kind of working ſhall be euidently ſeene in the examples following.

My neighbour mingled 12 *buſhels of fine vvheat worth* 14 *pence, vvith other corne, namely, wheate, barlie, and oates: the buſhell of wheat was worth* 18 *pence, the barlie* 11 *pence, the oates* 9 *pence, the buſhell of corne mingled together was worth* 10 *pence. The queſtion is, how much wheat? how much barlie? how much oates was mingled together?*

Here a part of the numbers ſought for is giuen, namely 12 buſhels, therefore firſt of all I counterchaunge the differences of the extremes as you ſee:

18 Wheate

18	Wheate	1
14	Fine wheate	1
11	Batley	1
10		
9	Oates	8, 4, 1 | 13

Then for the first terme of the proportion I take the difference attributed to that extreme whose measures are by number giuen, namely one bushell worth 14 pence, but for the third terme I take the nūber of the measures giuen, namely 12 bushels worth 14 pence a bushell. And so conclude the question propounded by two simple proportions. The first proportion is thus:

1 Bushell worth 14 pence requireth 13 bushels worth 9 pence the bushell: therefore 12 bushels at 14 pence the bushell require 156 bushels at nine pence the bushell.

The second is thus:

1 bushell worth 14 pence requireth 1 bushell of each other kinde: therefore 12 bushels at 14 pence the bushell require 12 bushels of ech other kind.

Wherefore with 12 bushels at 14 pence the bushell were mingled 156 bushels at 9 pence the bushell, and 12 bushell of each of the other kindes.

Suppose there were foure kindes of siluer: let a pound of the one be worth 20 *pound, of the other* 16 *pound, of the third* 14 *pound, of the fourth* 12 *pound. These foure kindes of siluer*

are to be mingled together, so that a pound of the mingled siluer be worth 15 *pound. Nowe there were taken* 33 *pound of the siluer worth* 16 *pound, the question is how many pound of each other kind is to be taken?*

Here a part of the numbers sought for is giuen, namely 33, therefore the differences being counterchaunged as you see:

	20	3
	16	1
15		
	14	1
	12	5

For the first terme of the proportion, I take the difference added to the extreme whose measures are by number giuen, namely one pound worth 16 pound, but for the third I take the nũber of the measures giuen, namely 33 pound at 16 pound the pound. Then I conclude the question propounded by three simple proportions, the first proportion is thus:

1 pound worth 16 pound, requireth one pound worth 14 pound: therefore 33 pound worth 16 pound the pound, require 33 pound worth 14 pound the pound.

The second is thus:

1 pound worth 16 pound, requireth three pound worth 20 pound the pound: therfore 33 pound worth 16 pound the pound

pound require 99 pound worth 20 pound the pound.

The third is thus:

1 pound worth 16 pound, requireth fiue pound worth 12 pound the pound: therfore 33 pound worth 16 pound the pound require 165 pound worth 12 pound the pound.

Therefore if there were 33 pound taken of the siluer worth 16 pound the pound, there might be taken 33 pound of the siluer worth 14 pound the pound, and 99 pound of the siluer worth 20 pound the pound, and 165 pound worth 12 pound the pound, I say they may be taken, but it is not needefull they should be taken. For seeing that here each number of the extremes is a number of multitude, therefore the alligation may be manifolde, according as I noted it before in the definition of prime alligation.

The prices of the same kinds of siluer being kept, if there had bene 6 pound taken of the siluer worth 16 pound the pound, and 8 pound of the siluer worth 12 pound the pound, that a pound of the mingled siluer might be worth 15 pound, you shall say thus:

1 pound worth 16 pound requireth 5 pound worth 12 pound: therefore 6 pound worth 16 pound the pound requireth 30 worth 12 pound the poūd.

And therefore vnto those 8 pound worth 12 pound the pound, there are to bee added

22 pound of the ſame price. The reſt is eaſie by that which hath beene ſaid before.

CHAP. XXV.

Of the ſecond kind of prime Alligation.

WE haue ſpoken of the firſt kind of prime alligation. Prime alligation of the ſecond kind is that, which by the meanes of the totall made of the numbers ſought for inferreth all the particulars by the helpe of proportion compounded by addition.

This compound proportion is commonly called the rule of Fellowſhip. In it for the firſt terme of proportion we take the totall made of the differences added together: for the ſecond, we take the totall made of the number of the meaſures, and for the antecedents of the reaſons remaining, the counterchannged differences of the extremes from the meane.

Mine hoaſt hath two ſorts of vvine, one worth ſixe pence a quart, another vvorth 12 *pence a quart. Of theſe two he purpoſeth to drawe ſixe quarts at* 10 *pence: how manie quartes therefore ſhall hee drawe of each ſeuerall kinde.*

Here the whole number giuen made of the numbers of the ſimple meaſures is ſixe. Therefore counterchaunging the differences as you ſee:

3
6 2

6 2

10

12 4

For the first terme I take the totall made of the differences which is 6. For the second terme I take 6, and for the antecedents of the reasons remayning I take 2 and 4, and then I make a compounded proportion in this manner:

2 2

6 6 therefore

4 4

Here therefore there must be drawen two quarts at 6 pence the quart, & 4 at 12 pence the quart. Hereby it appeareth that if the totall made of the numbers of the simple measures be equall to the totall made of the numbers of the differences, the numbers of the differences are the numbers of the simple measures.

Hiero King of Syracusa voweth to the goddes for the prosperous successe of his affaires a crowne of gold. Let vs suppose that Hiero for the making of this crowne gaue vnto a vvorkeman 500 pound of gold: and that the workeman made in deede a crowne of iust vvaight, but mingled some siluer in the crowne. The king demaundeth of Archimedes how much golde, how much siluer was in the mingled crowne.

For the resoluing of this question Archimedes tooke two lumpes of mettall of the same waight with the crown, but of the same

kind with the gold and ſiluer which was in the crowne. I ſay they muſt be of the ſame kind. For let there be two lumps of ſiluer of equall waight one to an other, but of vnequall fineneſſe, the finer lumpe ſhall fill a leſſer place then the other which is not ſo fine. The ſame is to be ſaid of the lumpe of gold. So that here will be a moſt manifeſt error, if the mettals be they either golde or ſiluer, be not both of one kind. I ſay therefore that the lumpe both of golde and ſiluer was of the ſame kinde with the golde and ſiluer in the crowne.

Now let there be 3 ſuch bodies choſen: let the one be a lumpe of gold, the other a lump of ſiluer, the third the crowne mingled of gold and ſiluer. And let them be hanged one after another ſeuerally in a veſſell full of water, and let the waight of the firſt be 968, of the ſecond 952, of the third 964. Then let the waights be taken for the value of the bodies themſelues: the value therefore of the lumpe of gold ſhall be 968, of the ſiluer 952, of the crowne 964. Now in the mingled crowne by ſuppoſition there are 500 pound waight: therefore here the totall example made of the numbers of the meaſures ſhall be 500, ſo that the differences being counterchaunged as you ſee:

968 12

964

952 4

For

For the first terme I take the totall made of the differences, namely 16, for the second I take the totall made of the numbers of the measures sought for, namely 500, for the antecedents of the reasons remaining, I take 12 and 4, and thereof frame a compound proportion in this manner:

		4	125
16	500: therefore		
		12	375

Whereby it is inferred that there was in the mingled crowne 125 pound of siluer, and 375 of gold.

Archimedes in Vitruuius is saide to haue found out the mixture of the golde, grounding his reason on the differences of the water running out of the vessell. But that the art may stand sure, I construe his manner of reason thus: that is to say, that by the knowen differences of the water running out of the vessell Archimedes attained vnto the other differences of the water vnknowen, and then at length answered the question by prime alligation of the second kind. Otherwise suppose an vnequall running out of the water, that is to say, when the gold was put in, say that there ranne out 20, when the siluer was put in 36, and when the crowne was put in 24. These seuerall quantities of water running out are to be taken for the values of the bodies as they were put in, according to the intent of him which gainesayeth mine asser-

tion. Therefore the value of a lumpe of gold of 500 pound, ſhall be leſſer then the value of a maſſe of ſiluer of the ſame waight, which is againſt reaſon. Therefore Archimedes by the knowen differences of the water running out, attained to the differences of the water remaining in this manner. Suppoſe that the veſſell into the which the crownes were put were 488 pintes. If therefore the water which ranne out when the lumpe of golde was put in were 20 pintes, the reſt was 468, the like we may iudge of the reſt.

My hoaſt mingled foure ſortes of vvine to the quantitie of 300 *quartes: A quart of the one vvine vvas vvorth* 12 *pence, of another* 10 *pence, of the third* 9 *pence, of the fourth* 7 *pence: He ſolde a quart of the mingled vvine for* 11 *pence, how many quartes of the firſt? how many of the ſecond? how many of the third? and how many of the fourth kinde did he mingle together?*

Here the totall number giuen made of the number of the ſimple meaſures is 300. Therefore counterchaunging the differences as you ſee:

7	1	
9	1	
10	1	
11		
12	4, 2, 1	7

For the firſt terme I take the totall made of

of the differences, namely 10, for the second I take 300, for the antecedents of the reasons remayning I take 1, 1, 1 and 7, and then I frame a compounded proportion thus:

		7	210
		1	30
10	300 therefore		
		1	30
		1	30

Whereupon I conclude that the quartes herein mingled, were first 210 at 12 pence a quart, then of each of the rest there were 30 quartes.

An Apothecary was to mingle pepper, sugar, cinnamon and ginger to the quantitie of fiftie ounces. An ounce of pepper vvas vvorth 25, sugar 24, cinnamon 22, ginger 18. An ounce of the spice mingled together vvas worth 23: How many ounces of pepper, how many of sugar, how many of each other kinde are to be mingled together?

Here first of all the totall giuen being made of the numbers of the simple measures is 500. Therefore the differences being counterchaunged as you see:

25	5
24	1
23	
22	1
18	2

For the first terme of the proportion I take the totall made of the differences, namely 9, for the second I take 500, for the antecedents of the reasons remaining I take 5, 1, 1 and 2, and thereof I frame a compounded proportion in this manner:

		5	277 $\frac{7}{9}$
		1	55 $\frac{5}{9}$
9	500 therefore		
		1	55 $\frac{5}{9}$
		2	111 $\frac{1}{9}$

Here therefore may be mingled of pepper 277 ounces and $\frac{7}{9}$, of sugar 55 $\frac{5}{9}$, and so much cinnamon, of ginger 111 and $\frac{1}{9}$. I say they may be mingled, but they shall not be mingled. For in this question each number of the extremes is a number of multitude, and therfore a manifold alligatiõ may be made therein, of which manifold alligation I gaue a note before in the first kind of prime alligation.

CHAP. XXVI.

Of second Alligation.

Hitherto of prime alligation: Second alligation followeth which by proportion cõpounded by multiplication and addition inferreth the meane.

Here both the numbers and prices are giuen of each seuerall measure, but there is but some part onely of the mingled measures giuen. Therefore here for the first terme I take

take the totall made of the numbers of the ſimple meaſures, and for the ſecond terme, I take the totall of the products made by multiplying thoſe numbers by their extremes, and laſt of all for the third terme of the proportion, I take the part giuen of the number of the mixt meaſures.

Of two kinds of corne there were mingled 10 *buſhels at* 16 *pence a buſhell, vvith* 18 *buſhels at* 12 *pence a buſhell: vvhat is the buſhell of mingled corne worth?*

Here firſt of all I adde together the numbers of the ſimple meaſures 10 and 18, the totall is 28. Then I multiply 10 by 16, and 18 by 12, and make 160 and 216, the totall of them is 376. The totall therefore of the prices of the ſimple meaſures is 376 pence. But alligation maketh the totall of the prices of the mixt meaſures equall to the totall of the ſimple meaſures as appeareth by the definition thereof. Therefore theſe 28 mingled buſhels ſhall be worth 376. Therfore I conclude the queſtion thus:

28 mingled buſhels are worth 376. therefore one buſhell of mingled corne is worth 13 3/7.

P. Ramus the moſt famous Philoſopher of our time, calleth this alligation, alligation of the meane ſought for: and defineth it to be that, which (two extremes being giuen) ſeeketh out the meane by diuiding the extrems

added together by their number. He addeth theſe wordes as if there be two extremes the diuiſor muſt be 2, if there be three extreames the diuiſor muſt be 3, and ſo foorth. Therefore by this definition if corne at 16 pence and 12 pence a buſhell be mingled together to the quantitie of 28 buſhels as is aforeſaide, the price of a buſhell of mingled corne ſhall be 14 pence, for here the nũber of the extremes is 2, and the extremes 16 and 12 being added make 28, ſo that if thou diuide 28 by 2, the quotient is 14.

But let vs ſee how true this is: Here by the rule of prime alligation the counterchaunging of the differences is in this manner:

16. 2

14

12 2.

By the which counterchange it is inſinuated, that when there are taken two buſhels of corne at 16 pence, then muſt there be two alſo taken of 12 pence the buſhell. Therefore when there ſhall be 10 taken at 16 pence the buſhell, then ſhall there alſo be 10 taken at 12 pence the buſhell. The numbers of the ſimple meaſures 10 and 18 being added together make 28. Therefore I reaſon thus: If when 10 buſhels at 16 pence the buſhell are mingled with 18 buſhels at 12 pence the buſhell, the meane is then 14: therefore when 28 buſhels of theſe ſimple ſeueral cornes are ſo mingled, that the price of a mingled buſhell

is

is 14 pence, then ſhall 10 buſhels at 16 pence the buſhell, be mingled with 18 buſhels at 12 pence the buſhell. (For it is all one way from Newhauſe to Heidelberge, and from Heidelberge to Newhauſe.) But this ſecond aſſertion is falſe (for here when 10 buſhels at 16 pence the buſhell are taken, then alſo there are taken 10 buſhels at 12 pence the buſhell as appeared before) therefore the firſt aſſertion is alſo falſe. Whether with 18, or with 10 buſhels at 12 pence a buſhell, you mingle 10 buſhels at 16 pence a buſhell, the extremes added together in both are 28, and their number is 2. Therefore whether you mingle 10 buſhels at 16 pence the buſhell, with 18, or 10 buſhels at 12 pence a buſhell, the price of a buſhell of mingled corne ſhall be all one, and ſo the price of the cheaper wheate ſhall be the price of the dearer wheate, which is abſurde.

I alwaies reuerenced my maſter while he was a liue as my duetie required, and much more doe I now imbrace the writings of that moſt holy Martyr being dead. But it is incident to a man to erre: and therefore he being a Philoſopher of moſt holy memorie, eſteemed thoſe men to be madde that thought it an hurtfull thing for a cõmon wealth to haue their faults amended: therefore following his owne decree by my definition, I correct his in this place. For he himſelfe either did correct it before his death, or if he had liued any

longer, I doubt not but he would willingly haue amended it: if he did correct it, truely it was neuer my chance as yet to see his correction.

Let 6 ounces of cloues at 36, and 8 ounces of cinnamon at 16, be mingled with 4 ounces of pepper at 15: how shall 4 ounces of the mingled spice be solde?

Here the numbers of the simple measures are 6, 8, and 4, whereof the totall is 18. The products of 36, 16, and 15, made by multiplying them by 6, 8, and 4 are 216, 128, and 60. The totall of them is 404. Therefore I will say thus:

18 ounces of mingled spice are worth 404
therefore 4 ounces are worth $89\frac{7}{9}$.

Let the prices of the simple spices be as they were before: and let $\frac{1}{2}$ of an ounce of cloues, and $\frac{1}{3}$ of an ounce of cinnamon be mingled with $\frac{1}{4}$ of an ounce of pepper: vvhat shall an ounce of the mingled spice be worth?

Here first of all the vnlike measures which are giuen $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ must be made like. For the propertie of generall alligation requireth that the measures should be like. Therefore by reduction of fractions to one denomination, for $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, I finde out $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$. Then that the worke may be more easie for the parts found out, by reduction of fractions to whole numbers, I take 6, 4, and 3. The totall therefore of the numbers of the measures shall be 13. Then multiply 36, 16, &

15,

15, by the numbers of their measures 6, 4, and 3, you shall produce 216, 64, and 45: whereof the totall is 325. Then shall you conclude the question thus:

13 ounces of mingled spice are worth 325, therefore one ounce is worth 25 $\frac{5}{13}$.

CHAP. XXVII.

Of Manifolde proportion continued in the termes.

MAnifolde proportion compounded in the termes hath beene handled hitherto: that remayneth vvhich is continued in the termes.

Manifold proportion continued in the termes is, vvhen vnto the disiunct termes of the reasons giuen, other proportionall termes doe aunswere, vvhereof each middlemost terme ioyneth the antecedent reason with the consequent.

As in the examples following

2 3 4 5
8 12 15

For the reasons giuen in the vppermost rowe of numbers (namely a subsesquialtera, and a subsesquiquarta) are in the nethermost row of numbers so continued and knit one to an other, that of the two middlemost termes there is made one, which according to the reasons giuen in the first ranke of nũbers is both the consequent of the former, and the antecedent of the latter. For as 2 is

vnto 3, so is 8 vnto 12: and as 4 is vnto 5, so is 12 vnto 15.

In this kind of proportion therefore there is intended an inuention or finding out of the least termes continually proportionall vvith the reasons giuen how many so euer.

This inuention is set foorth in the 4 prop. 8, where it is demonstrated by what meanes, how many reasons so euer being giuen in the least termes, you may finde out other least termes continually proportionall, keeping the reasons giuen. Let that probleme therefore be turned into two theoremes: whereof let the one concerne two reasons giuen, and let the other concerne many reasons giuen in this manner:

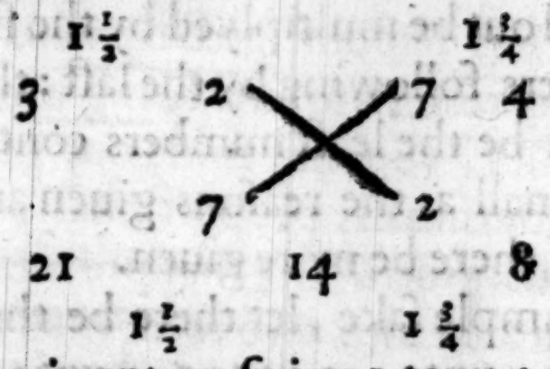
I

If the middlemost termes being prime one to another of the two reasons giuen, be set crossewaves, and then the former of the two multiply the termes of the first reason, and the latter the termes of the consequent: the three products shall be the least termes continually proportionall as the reasons giuen are.

Therefore we are to note, that in the reasons giuen the middlemost termes are either prime numbers or compound, if they be cõpound, they must be reduced to prime numbers. As in 3, 2, 7, 4, the middlemost numbers are prime numbers in respect one of an other. Wherefore being set crossewise one

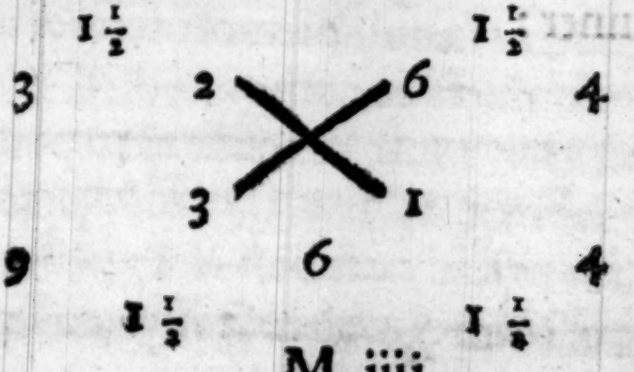
vnder

vnder another, if you multiply 7 by 2 and 3, which are the termes of the former reaſon: and then multiply 2 by 7 and 4, which are the termes of the reaſon following, the products 21,14,8, ſhall be the leaſt numbers continually proportionall, as the numbers giuen are, as you ſee here:



For as 3 is vnto 2, ſo is 21 to 14: and as 7 is to 4, ſo is 14 to 8.

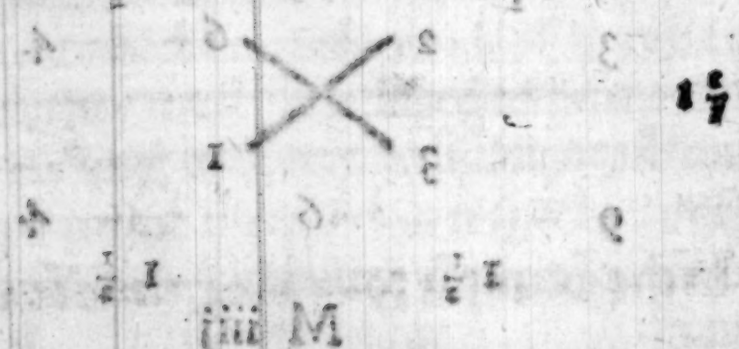
Let theſe two reaſons or proportions be giuen like one to an other 3, 2, 6, 4. Here 2 and 6 are compound numbers in reſpect one of another, wherfore I reduce them to prime or the leaſt numbers, namely to 1 and 3, and place them croſſewiſe, and multiply 3 into 3 and 2 the antecedents, and likewiſe 1 into 6 and 4 the conſequents, the products are 9,6, 4, which are the leaſt termes continually proportionall, as the reaſons giuen, according as you ſee them here written.

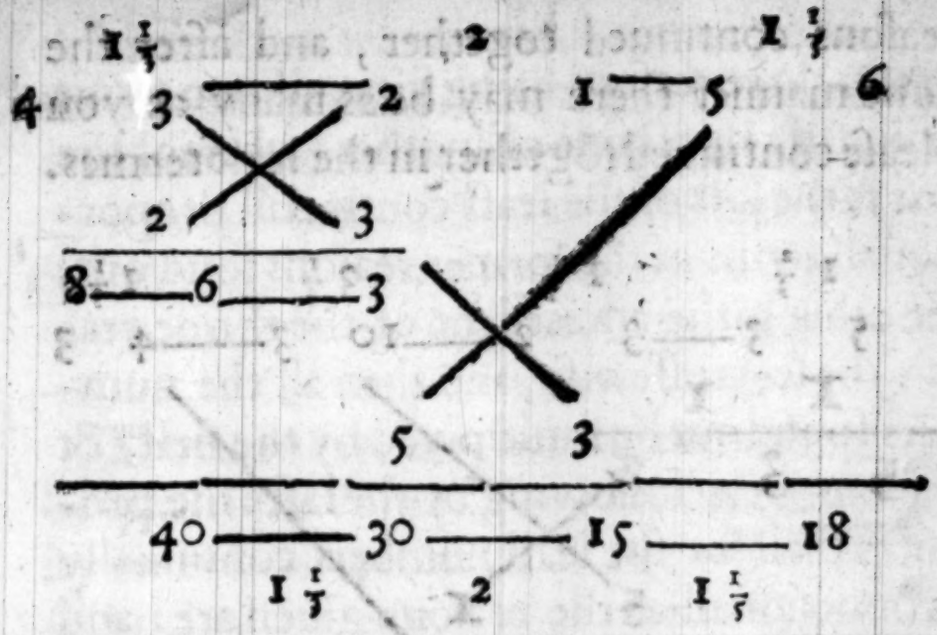


II.

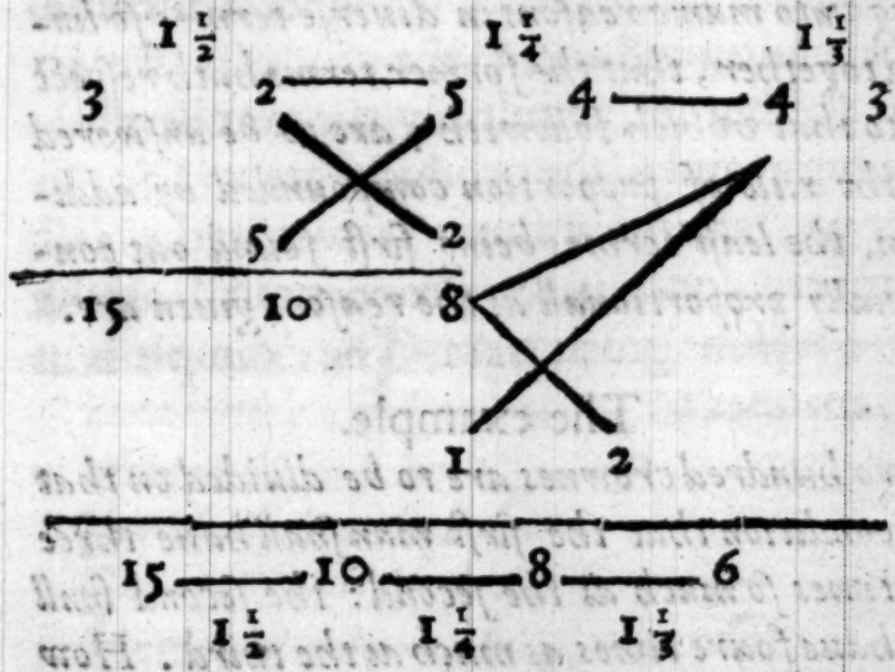
Any three reaſons being giuen, if two terms prime in reſpect one of another, (whereof the one is the laſt of the leaſt continuall proportionall terms of the former reaſons foūd out, the other is the antecedent of the thirde reaſon) be ſet croſſewiſe, and then all the numbers found out be multiplyed by the firſt, & the numbers following by the laſt: the products ſhall be the leaſt numbers continually proportionall as the reaſons giuen are: and ſo forth if there be more giuen.

For example ſake, let there be three reaſons giuen 4 vnto 3, 2 vnto 1, 5 vnto 6, to be continued in the leaſt or prime proportionall termes. Firſt therefore I diſpatch the former two reaſons, and find 8, 6, 3. And becauſe 3 (the laſt of them that are found out) is a prime number to 5 the antecedent of the reaſon remaining: therefore they being ſet croſſewiſe, let 5 multiply 8 and 6, the products are 40 and 30: but let 3 multiply the numbers following 5 and 6, the products are 15, 30, ſo that we haue now foure termes in the leaſt numbers continually proportionall as the three reaſons giuen were. The example is in this manner:



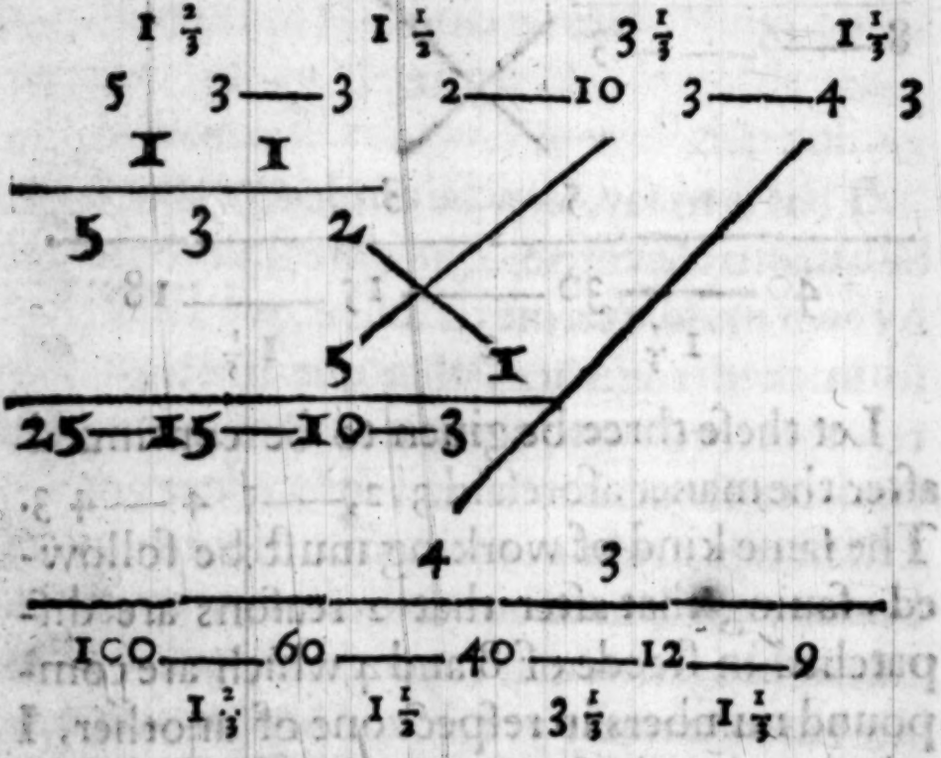


Let theſe three be giuen to be continued after the maner aforeſaid, 3 2, —— 5 4, —— 4 3. The ſame kind of working muſt be followed, ſauing that after that 2 reaſons are diſpatched, in ſteede of 8 and 4 which are compound numbers in reſpect one of another, I take 2 and 1, which are prime numbers as you ſee in this example:



In the example following there are foure

reasons continued together, and after the same manner there may be as many as you please continued together in the least termes.



Therefore the questions of proportion wherein any number offereth it selfe to be parted according vnto manie reasons in diuerse termes so linked together, that the former terme hath respect vnto that vvhich followeth, are to be answered by the rule of proportion compounded by addition, the least termes being first found out continually proportionall as the reasons giuen are.

The example.

Two hundred crownes are to be diuided on that condition that the first man shall haue three times so much as the second: the second shall haue foure times as much as the third. How much shall euery man haue?

Here

Here there are two reasons propounded, the one is triple, the other is quadruple, which are knit together in this manner:

3	1	
a	c	
	c	e
	4	1

That is to say, let a be triple to c, but let c be quadruple vnto e. These are knit together by two middle termes 1 and 4, which being so lincked together, that one and the same terme according to the reasons giuen, may be both consequent to a, and antecedent vnto e, I shall easily resolue the doubt. Now this will be done, if I finde out the least termes continually proportionall as the termes are which are giuen, namely 3, 1, 4, 1, the which termes found out will be these, 12. 4. 1. For then they being added together, the proportion will be inferred thus:

	12	141 $\frac{3}{17}$
17	200 therefore	
	4	47 $\frac{1}{17}$
	1	11 $\frac{11}{17}$

Let there be 24 crownes so to be parted, that as often as the first hath 3, so often the second hath 4, and as often as he hath two, so often the third hath 3, and as often as the third hath 6, so often the fourth hath 1. What shall each mans portion be?

Here the termes of the three reasons 3, 4: 2, 3, 6, 1: are to be reduced (by the last theoreme)

into foure termes continually proportionall as the reaſons giuen are, the which foure termes are theſe, 3, 4, 6, 1. The totall made of theſe termes added together, ſhall be the firſt terme of the proportion, and being ſeuerally ſet downe, they ſhall be the third terme. The number which is to be diuided ſhall be the middlemoſt, and then the proportion ſhall be concluded thus:

	3	$5\frac{1}{7}$
	4	$6\frac{6}{7}$
14	24: therefore	
	6	$10\frac{2}{7}$
	1	$1\frac{5}{7}$

Foure other men vvere to diuide 120 *crownes gotten in partnerſhippe, ſo that as often as the ſecond had* 5, *ſo often the third had* 9, *and as often as the third had* 7, *ſo often the fourth had* 11, *and as often as the fourth had* 9, *ſo often the firſt had* 13, *vvhat vvas each man his portion.*

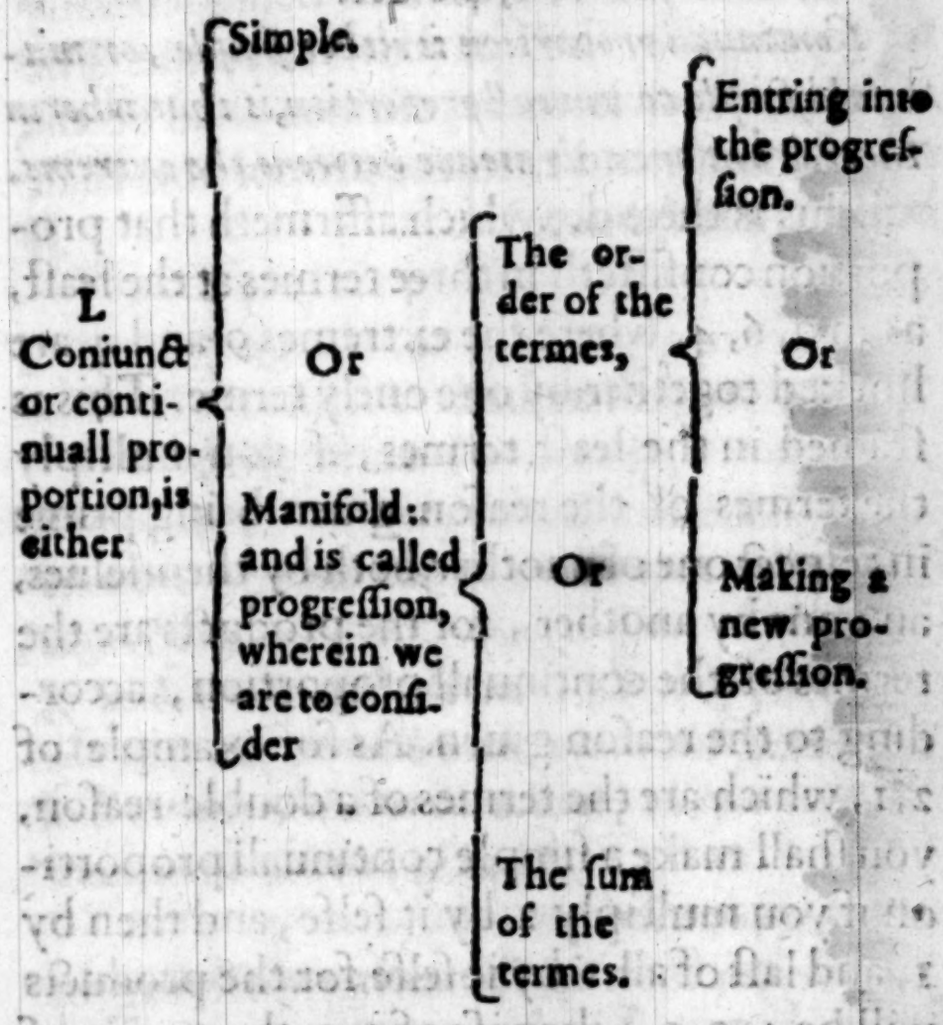
This queſtion is ſomewhat differing in ſhewe, but the working is all one. For the matter is diſpatched by proportion of addition, the leaſt numbers being found out continually proportionall as the reaſons giuen are. As you ſee here:

	35	$12\frac{6}{17}$
	63	$22\frac{4}{17}$
340	120 therefore	
	99	$34\frac{16}{17}$
	143	$50\frac{8}{17}$.

CHAP.

CHAP. XXVIII.

Of continuall proportion.



Hitherto of disiunct proportion: now followeth continuall.

Continuall proportion is that whose middle termes doe all supply the place of an antecedent, and a consequent.

In continuall proportion the first extreme is onely an antecedent, and the last is onely a consequent: of the other termes, which are in the middest betweene the extremes any of

them in respect of that which went before it is a consequent, in respect of that which followeth it is an antecedent, as this is in three termes 9, 6, 4, in foure termes, 8, 12, 18, 27, in fiue termes 32, 16, 8, 4, 2 &c.

Continuall proportion is either simple, or manifold, simple continuall proportion, is that wherin there is but one only meane betwene the extrems.

This is the 9 d. 5. which affirmeth that proportion consisteth in three termes at the least, as in 9, 6, 4, where the extremes 9 and 4 are lincked together by one onely terme. This is framed in the least termes, if you multiply the termes of the reason giuen being prime in respect one of another, both by themselues, and one by another, for the products are the termes of the continuall proportion, according to the reason giuen. As for example, of 2, 1, which are the termes of a double reason, you shall make a simple continuall proportion if you multiply 2 by it selfe, and then by 1, and last of all 1 by it selfe, for the products will be 4, 2, 1. Likewise of 3, 2, the termes of a sesquialter reason, you shall by the same meanes make 9, 6, 4.

The propertie of it is this: The product made of the extremes, is equall to the product made of the meane, and contrariwise.

This is the 17 p. 6. concerning magnitudes, and also the 20 p. 7. where this propertie is also attributed vnto nūbers, because in three proportionall numbers, the product made of the

the extremes, is equall vnto the ſquare made of the meane. As in 9, 6, 4, the product made of 9 and 4, is 36, which is equall to the ſquare of the meane, namely vnto ſixe times ſixe, or which is all one to the product made of the meane multiplied by it ſelfe. For it is, as if you ſhold inferre the proportion diſiunctiuely in foure termes after this manner:

9 giue 6: therefore 6 giue 4.

Wherefore it is to be gathered, that this propertie is generall to all ſimple proportion whether it conſiſt of three or foure termes.

Manifold continuall proportion, is that wherin many meane termes proceede continually after one and the ſame reaſon. Herenpon it is called progreſſion.

As 1, 2, 4, 8. Item 16, 24, 36, 54, 81.

In progreſſion we haue an eye to the finding out of the order of the terms making the progreſſion, or elſe to the ſumme of them.

The finding out of the termes is two fold: for either the terms giuen enter into the progreſſion, or elſe new termes are made by their meanes.

All the termes of a proportion giuen, can not be extended and continued. In manifold proportion it may be, but in other reaſons it can not ſo well be done. It ſhall appeare therefore by the theoreme following, what termes giuen will admit a progreſſion.

If the laſt terme ſauing one doth diuide the product made of the laſt terme, the quotient ſhall be the proportionall terme following,

Or, If the laſt terme doth diuide the product made of the laſt ſauing one, the quotient ſhall be the antecedent proportionall.

This Theoreme is gathered out of the 18 p.9. It is alſo deduced as it were a cõſequence out of the proprietie of continuall proportion, whereby not onely proportion, but any reaſon giuen may be continued. As in 3,9, 27, wherein 9 diuideth 729, the product made of the laſt number 27, the quotient is 81, which is the fourth proportionall number. Likewiſe let 9 which is the ſecond number diuide 9 which is the product made of the firſt number 3, the quotient 1, is the firſt proportionall number. So that the progreſſion ſhall be in this manner, 1, 3, 9, 27, 81. This continuation of the termes in manifold proportion may be infinitely inferred onely by the multiplication of the laſt terme by the denominator of the reaſon or proportion:

Yet in thoſe whoſe reaſon is manifold, any terme ſought for may be fouud out readily in this manner:

If you diuide the product made of the laſt number multiplied in it ſelfe, by any of the numbers which went before it: by how many degrees the diuiſor went before the number multiplied, by ſo many degrees ſhall the quotient followe the ſame number multiplied.

For example ſake, let this double proportion be giuen.

2	4	8	16	32	64	128
4	3	2	1	2	3	4

Antecedents Consequents.

And let 16 be the number to be multiplied by it selfe, the product is 256, diuide that by 8 which is the number next before it, the quotient 32 is the number which must followe next after 16. But if you diuide the product 256 by 4 which went before 16 in the thirde place, the quotient will be 64, which must follow in the third place after 16. If you make the diuision by 2, which went before 16 in the fourth place, the quotient 128 must follow in the fourth place after 16, and so forth in the rest. For it is as if you should set the rule of proportion thus:

If 2 giue 16: then 16 shall giue 128.

Out of the foresaid abridgement, another ariseth, whereby we may finde out any terme of manifold progression, in this manner:

If some certaine termes of continuall progression be giuen, and the numbers following one another from an vnitie according to their naturall order, doe aunswere the saide termes from the second forward, the quotient of the product made by any of the termes whatsoeuer diuided by the first terme, shall be the terme of the progression more by one, then both the numbers answering to the numbers multiplied, doe amount vnto being added together.

As in this triple proportion, let there be giuen ſome termes, and let the numbers be written vnderneath them in order as you ſee:

3	9	27	81	243	729
0	1	2	3	4	5

Then if you multiply 27 by 81, and diuide the product 2187 by 3, which is the firſt nũber in the progreſſion, the quotient 729 ſhall be the ſixth terme of the progreſſion which is more by one, then 2 and 3 (that is to ſay 5) are. Likewiſe if you multiply 243 by it ſelfe, and diuide the product by the firſt, the quotient ſhall be 19683, to be placed in the ninth place, and ſo forth of the reſt.

The termes aſwell of manifolde reaſon as of proportion and progreſſion, may in this manner be continued infinitely. But the nature of numbers doth not beare it ſo well in the other kinds of reaſon or proportion. For firſt of all, if of the reaſon giuen the termes be prime numbers one to another, then can not a third, much leſſe a fourth or fifth in whole numbers be adioyned to them in continuall proportion: as it appeareth by the 16 p. of the 9. As in the reaſon of 3 to 5, which is ſuperpartient two fifts, you ſhall neuer bring forth a continuall proportion, vnleſſe you ioyne to it this mixt or ſurde number $8\frac{1}{3}$, which cannot be expreſſed by an whole nũber. Likewiſe how many proportionall numbers

bers ſoeuer being giuen, if the extremes be prime one to another, no ſuch number can be giuen at the laſt, as the ſecond is vnto the firſt, by the 17 p. of the 9. As in this ſubſeſquialter 4, 6, 9, becauſe 4 and 9 are prime one to another, therefore there can be no whole number in ſuch proportionalitie vnto 9, as 4 is vnto 6. Therefore in ſuch kind of proportions this Theoreme following taketh place.

If the termes of continuall ſimple proportion, not admitting an vſuall progreſſion be multiplied by the antecedent of the reaſon whereof they conſiſt, and the laſt be multiplied by his conſequent: there ſhall be produced foure leaſt termes continually proportionall according to the numbers giuen: and ſo forth continually by multiplying the products by the antecedent giuen, and the laſt by the conſequent giuen, you ſhall find the leaſt termes how many ſo euer continually proportionall according to the reaſons giuen:

As for example, ſeeing that this ſeſquitertia 16, 12, 9, hath not a fourth number in progreſſion. Therefore firſt of all, I multiply all the numbers giuen by 4, the antecedent of the reaſon giuen betweene 4 and 3, whereof they were made: the products ſhall be 64, 48, 36. Likewiſe I multiply the laſt number 9 by 3 the conſequent giuen, the product is 27, the which numbers are all continually proportionall next vnto the numbers giuen.

After this manner many others may be found out.

					The termes of the
		4	3		reaſon giuen.
	16	12	9		The ſimple prop.
64	48	36	27		The firſt pro-
256	192	144	108	81	greſſion.

Behold alſo this example following of a manifold ſuperparticular reaſon.

		5	2	
	25	10	4	
125	50	20	8	
625	250	100	40	16

Thus much concerning the ranging of the termes: now followeth the ſumme of the progreſſion.

The finding out of the ſumme, is that which (the firſt terme of the progreſſion increaſing being ſubducted from the ſecond and the laſt) addeth vnto the laſt terme giuen a number, to the which the remainder of the laſt terme hath ſuch proportion as the remainder of the ſecond terme hath vnto the firſt.

I ſaid (the progreſſion increaſing) that ye may vnderſtande that the firſt terme is here taken for the leaſt, and the laſt for the greateſt. Now the maſterie of this inuention is generall, being not onely of force in manifold proportion, as that rule is which the cõmon ſort of Arithmeticians do preſcribe, but all progreſſion of what proportionalitie ſo

euer

euer giuen: It is taken out of the 33 prop. 9. which affirmeth this, of how many proportionall numbers soeuer following one another. If from the second and the last terme there be taken away numbers equall to the first, the remainder of the last terme shall be vnto all the antecedents going before it, as the remainder of the second terme is vnto the first terme. Therefore these three proportionall termes being found out, you shall set the excesse or the remainder of the last terme in the third place, and the remainder of the second in the first place, and the first or least terme of the progression in the middlemost place, and then worke by the rule of three, whereby you shall inferre a number contayning all them which were antecedents to the last, which being therefore added to the last shall containe the summe of them all.

As in this manifold progression, 2, 4, 8, 16, 32, if 2 be taken from 4, and 32, there remaine 2 and 30. Now as 2 the remainder of the second terme is vnto the first terme 2, so is 30 vnto all the antecedents. Therefore for the working of the rule, the termes shall stand thus:

2 giue 2: therefore 30 giue 30.

The reason is in ech place alike and equall. Therefore seeing that the totall made of the numbers going before the last is 30, it being added to the last, namely vnto 32, declareth the summe of the progression to be 62.

Likewiſe in this ſubtriple progreſſion, 2, 6, 18, 54, let 2 be taken from 6 and 54, the remainders ſhall be 4 and 52. Now as 4 the remainder of the ſecond terme is vnto the firſt terme 2, ſo is 52 the remainder of the laſt terme vnto all the antecedents. Wherefore the three termes knowne ſhall be thus ordered for the finding out of the fourth terme.

4 2: therefore 52 26

And for ſo much as this fourth number, is equall vnto all the antecedents, it being added to the laſt terme 54, the totall 80 ſhall be the ſumme of the progreſſion.

Likewiſe let there be this ſubſeſquialter progreſſion, 16, 24, 36, 54, 81. The remainders of the ſecond, and laſt terme are 8, and 65, whereupon by theſe three proportionall numbers 8, 16, 65, the fourth number 130 is inferred, which is the totall made of all the antecedents except the laſt, it being therfore added vnto the laſt terme 81, declareth the ſumme of the progreſſion to be 211. Whereby it appeareth that the firſt, ſecond, and laſt terme of the progreſſion being knowen the ſumme cannot lie hid.

An Example.

A certaine man ſolde his houſe to be payde for it in wheate after ſuch a manner, that vvhen he that bought it came in at the firſt doore he ſhould giue him one graine, at the ſecond two, at the third 4, and ſo foorth proceeding continually

nually by double proportion according to the number of the doores. He that ſhrunke from his bargaine ſhould pay to the other 12 *crowns for a penaltie. Now being* 60 *doores, the queſtion is how much wheat he was to pay.*

A rich man of Baſill not ignorant of the huge increaſe of Geometricall progreſſion, ſold his houſe ſometimes vppon this condition being among his cuppes. But the condition of his bargaine being brought vnto an account, the number of the wheat was found to be vnmeaſurable, ſo that all the houſes of Baſill being turned in garners, were not ſufficient to receiue it. I thought good to ſet downe the example. Let the 60 terme of the progreſſion be compendiouſly found out.

1	2	4	8	16	32	512	1024
0	1	2	3	4	5	9	10
					524288		19
				536870912			29
			2882303761	517711744			58
			5764607523	034234488			59

Wherefore the ſumme of all the graines ſhall be,

115292150460684697 5

The which number is ſo great, that it ſeemeth ſufficient to match the ſand of the Adriatique ſea.

FINIS.

A BRIEFE TREATISE ADDED CONCERNING THE RESOlution of the Square and Cube in numbers, whereby the side of them may be found.

A Figured number, is a number made by the multiplying of one number by another.

The sides of the figured number, are the numbers by vvhose multiplication it was made.

Albeit a number be not a magnitude admitting figure and angles, yet seeing that the multiplying of numbers is like vnto the making of right lined plane figures, therefore for that resemblance of a Geometricall thing, the names of figures are attributed vnto nũbers produced by multiplication. For euen as a right line drawen first a long one right line comprehendeth a plane figure, and then a long another, which is the depth, comprehendeth a bodie: euen so the multiplying of one nũber by another, maketh a plane number (so called becauſe the vnities whereof it

consisteth

consisteth, may be contriued into a plane figure) and that being multiplyed by a third number maketh a solide number, receiuing that title because it may be fashioned like a solide figure. The numbers multiplyed one by another represent their sides.

A figured number is twofold, a plane and a solide number.

A plane number is that, which is made by multiplying two numbers one by another.

16 d. 7. Multiplication (as is aforesaide) maketh a certaine thing like to a right angled parallelogramme. Wherefore here are excluded all other numbers, which others call trigonals, pentagonals, &c. made by addition, whereof Euclids Arithmeticke maketh no mention.

A plane number is either made of two equall numbers as a Square: or of two vnequall numbers, as an Oblong.

18 d. 7. A square number is made by a nũber multiplied by it selfe, as foure times 4 are 16, fiuetimes 5 are 25, ten times 10 are 100: for these products may by their seuerall vnities be so displayed in a plane surface, that they may represent plane squares, as here you see in the nũber 16: 1 1 1 1 | 16 is the
wherefore by the ma- 1 1 1 1 | platforme,
king of this Square, it 1 1 1 1 | 4 is the
is euident that a square 1 1 1 1 | side.
may be made of any number giuen, and that any number may be the side of a square. But

as it is requiſite that the ſides (being leſſer then 10) of the ſquares vnder an hundred ſhould be gathered by the Table of multiplication: ſo the ſides of greater Squares are to be ſought out by art. The ſquares whoſe ſides are ſimple numbers, are here ſet downe as you ſee:

The ſides	1	2	3	4	5	6	7	8	9
The ſquares	1	4	9	16	25	36	49	64	81

The Square is knowne by finding out his ſide expreſſed by a whole number.

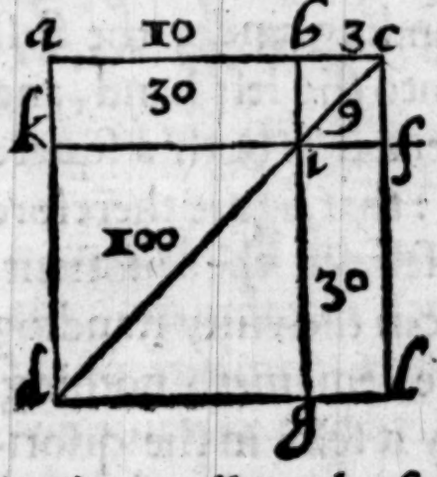
Albeit the finding out of the ſide of a ſquare, be applied to each number giuen as to a ſquare, yet the ſquare numbers onely haue a ſide to be expreſſed by a certaine nũber of vnities or by rationall numbers, the other are to be expreſſed but in power only. The ſides are commonly called rootes by a metaphoricall phraſe.

The ſide of a ſquare is to be found by the Theoreme following.

If the odde degrees of a ſquare nũber being marked from the right toward the left hande with points, you ſubduct frõ the number giuen, the particular ſquare of the laſt period, ſetting the ſide therof aloñe by it ſelfe, 2 then going on if you diuide the remainder (if ther be any) with the figure going before it, by the double of the ſide ſet alone by it ſelfe, 3 and multiply the quotient found out (being placed by the ſide which was firſt ſet alone by it

ſelfe,

ſelfe, and alſo before the doubled number on the right hand) by both the numbers (namely by the doubled number, and the figure ſet by it) being counted as one diuiſor, ſubducting the products from the number giuen, and then renew this laſt worke of diuiſion ſo many times as there are prickes remaining, the ſide of the ſquare ſhalbe found out.



This artificiall deuiſe is borowed out of the 4 p. 2. Where by demonſtration it is proued, that if a right line be cut into two ſegments, how ſoeuer, the ſquare of the whole line is equall to the ſquares of the ſegments, and to the two right angled figures made of the ſegments: as in the figure here annexed, the two Diagonals k g, and b f, are the ſquares of the ſegments a b, and a c. Alſo the complements b k and f g, are the right-angled figures made by multiplying the line a b by b c.

The ſelfe ſame parts are to be found in any ſquare number. As for example in the number 169, whoſe ſide is 13. This ſide being diuided into two peeces 10 and 3, multiply ech peece by it ſelfe once, namely 10 by 10, and 3 by 3, thē multiply one by another, as 10 by 3, & 3 by 10, ſo ſhall you haue 4 plane numbers, whereof two are Squares as here you ſee.

10 3
10 3
———
100
30
30
9
———
169

Therefore as the Square 169 is made by the adding together of theſe foure plane numbes, ſo by ſubducting them ſeuerally it is reſolued. Firſt therefore I marke each odde place with points. becauſe the particular ſquares are to be found in the odde places. Then for ſo much as the vnitie ſtanding vnder the firſt point next vnto the left hand, and repreſenting the laſt period, is both a ſquare and the ſide of a ſquare: that figure therefore being ſet alone by it ſelfe in the quotient, and being ſubducted from the vnity ſtanding ouer the point, there remaineth nothing. This vnitie ſet alone by it ſelfe in the quotient ſhall ſignifie 10, when another figure is ſet by it repreſenting the ſide of ſome other particular ſquare. Whereupon I ſay, that the greater Diagonall k g, is nowe ſubducted from the whole ſquare, and the ſide of it k i or a b (for they are equall one to another) and alſo the ſide of one of the complements is found out. This is the firſt ſteppe to this reſolution.

Moreouer I double the figure found out, becauſe being doubled, it is the ſide of both the complements taken ioyntly together, namely k i and g i. Then ſetting 2 the doubled number vnder 6, I diuide 6, (which in this place is as much as 60, and repreſenteth both the complements) by 2, the quotient is

3, 10

3, repreſenting the other ſide remaining of the complemēt, namely, i f, or b c, the which number I ſet in the quotient, and count it for the ſegment remaining of the right line giuen. Wherefore becauſe this number 3 is the ſide of the Diagonall remayning, that is to ſay, of the leſſer ſquare b f: therefore being ſet by the diuiſor on the right hand, and multiplyed both by it ſelfe, and by the diuiſor, it bringeth forth three plane numbers, namely, the ſquare b f, and the two complements a i and i l, which being ſubducted from the nūbers ſtanding ouer them, there remaineth nothing. The example is thus:

1	6	9	(13 which is all one as if you had ſet down the numbers found out thus:	1	6 9
1	2	3		1 0 0	The greater Diagonall.
		3		6 0	The two Cō plements.
6	9			9	The leſſer Diagonall.
				1 6 9	

Take this for an other example to make this deuiſe more plaine. Let the Square giuen be 1764. This number being marked with two points telleth vs that the ſide thereof is to be written with two figures. Firſt therfore beginning at the point on the left hande, I ſeeke the ſide of the laſt periode, namely of 17. But for ſo much as it is no ſquare number, I take 4 the ſide of the next leſſer ſquare, which I ſet alone by it ſelfe in the quotient, and then multiply it by it ſelfe, the product

is 16, which being ſubducted from 17 there remaineth 1. Moreouer, I double the ſide found out, the product is 8, I place this doubled number vnder 6, and by it I diuide 16 ſtanding aboue it, the quotient is 2, which muſt be ſet by 4. This quotient 2 muſt be ſet before the diuiſor 8 on the right hand vnder the point, and then muſt it be multiplied both into it ſelfe and into 8, the product is 164, which being ſubducted from the figures ſtanding ouer them, there remaineth nothing, whereby I gather that the number giuen is a iuſt Square. The example ſtandeth thus:

```
      1
    1 7 6 4 (42
  ---------
    1 6 8 2
            2
      1 6 4
```

The ſame manner of working is to be followed in greater ſquare numbers giuen, ſauing that the former part of the worke is to be vſed but once, but the latter part is to be followed ſo many times as there are pointes remaining, excepting the laſt. As in $\dot{5}4\dot{7}5\dot{6}$, I ſay, that the ſide of the ſquare next vnto 5 is 2, therefore 2 being ſet in the quotient, and multiplied by it ſelfe make 4, and taken from 5, the remainder is one. Moreouer I double the quotient, the product is 4, which I ſet vnder the next figure toward the right hand, and

and thereby diuide 14, the quotient is 3, the which 3 being ſet both in the quotient, and alſo before the diuiſor toward the right hand, I multiply both the numbers by it, the product is 129, this being ſubducted from 147 ſtanding aboue it, the remainder is 18. But becauſe there is yet one point remaining with the which I haue not medled, therfore againe I double all the whole quotient. For in this caſe I muſt take 23 for the one ſide of one former ſquare, and generally in great numbers when I light vppon more particular ſquares then two, I muſt eſteeme them but as two, and take the ſides which are firſt found out but as the ſides of one only ſquare. Therefore twiſe 23 are 46, by this I diuide 185, the number to be ſet in the quotient is 4, the which number alſo muſt bee ſet before the diuiſor on the right hande, then muſt 464 be multiplied by 4, the product is 1856, this product beeing ſubducted from the numbers ſtanding ouer it, there remaineth nothing. The example ſtandeth thus:

```
1 1 8
5 4 7 5 6 (234
-----------
4 4 3 6 4
1 2 9 5 6
    4
  1 8
```

See also the example following:

1 0 9 4 2 8 6 4 (3308

Out of this inuention this conclusion followeth:

The number whose side cannot be expressed by whole numbers, is no square number.

Such are all prime numbers, & (the squares themselues excepted) all other compound numbers. For if in them you desire to finde out the square side, you shall labour in vaine, because they are not squares for to the whole numbers arising in the quotient, there will be some fraction adioyned, whereby it commeth to passe, that the number of the side is not to be expressed by a true number, and commonly it is called a surde number.

Yet if you adioyne to the side found out the number remaining, taking his denomination from the double of the side augmented by an vnitie, you shall finde the next side that may be like to the side of a square.

As if from 40 you take the neareſt square, namely 36, the remainder is 4. Here therefore the side sought for of the square, exceedeth not the side found out by an vnitie, but either by one, or more parts of some whole number: wherefore I double 6 the side found out, and adde an vnitie to it being doubled, the totall is 13, this number I set vnder 4 the remainder, and say that the side of 40 sought as neare as may be, is $6\frac{4}{13}$, the denominator of the fraction being added to the greateſt

square

ſquare in the number giuen, namely vnto 36, maketh the next greateſt ſquare aboue it, namely 49, whoſe ſide is 7. But this ſurde ſide namely $6\frac{4}{13}$, multiplyed by it ſelfe maketh $39\frac{133}{169}$, which are not iuſt equall vnto 40 the number giuen. The like iudgement may be vſed in the reſt which are not ſquares.

Thus much concerning plane numbers, but eſpecially ſuch as are ſquare numbers. The ſolide number followeth, which is a number made of three numbers.

Soll. number

17 d. 7. A ſolide number is made by a twofold multiplication, euen as a ſolide bodie among magnitudes is made by a twofold cõprehenſion: the firſt is like that which maketh a plane figure, the ſecond is like that which maketh a body. As three times 4 taken fiue times: for three times 4 are 12, this is a plane number, fiue times 12 are 60, this is a ſolide number.

A ſolide number is made either of 3 equall numbers as a Cube: or of 3 vnequall numbers as an Oblong.

19 d. 7. A cube in numbers is made by multiplying the ſide thereof by the ſquare of the ſide: whereby we may gather, that to multiply a number cubically, is to multiply the nũber made of it ſelfe, by himſelfe againe as foure times 4 foure times are 64, which is a cube hauing relation to a Geometrical thing, partly becauſe their making is like, partly alſo becauſe the vnities of the number ſet in

a certaine diſtance one from another reſemble a cube.

The cube is knowne by finding out the ſide expreſſed by a whole number.

For euery Cube in numbers hath ſuch a ſide as may be expreſſed by whole numbers, but in magnitudes it is not alwayes ſo, as in deede in magnitudes there are many things not to be expreſſed by whole numbers. Now for ſo much as the ſide of any Cube vnder 1000 is a ſimple figure, it is meete before we vndertake to find out the ſide of any greater number, that we ſhould know what Cube is made of each ſimple figure, and what is the ſide of any Cube leſſer then 1000, as I haue here ſet them downe.

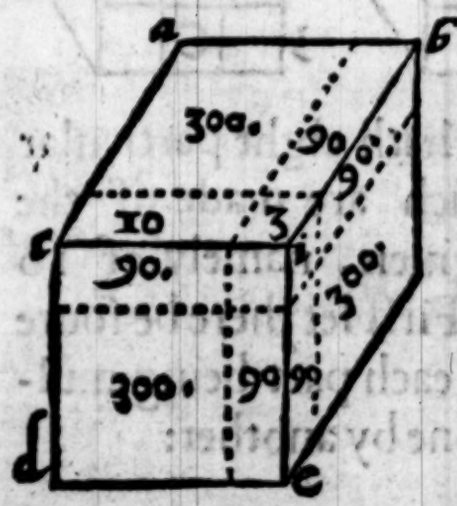
1	2	3	4	5	6	7	8	9
1	4	9	16	25	36	49	64	81
1	8	27	64	125	216	343	512	729

But in ſearching out of greater Cubes, we muſt proceede as the Theoreme following teacheth vs.

If you diſtinguiſh with pointes as it were into periods, the Cube giuen beginning at the firſt figure on the right hand, and omitting ech two figures continually, and firſt of all ſubduct the particular Cube of the laſt period from the number giuen, ſetting the ſide therof in the quotient: and then ſet the triple of the quotient vnder the figure next following the former point on the right hande, and the ſquare of the quotient being tripled beneath

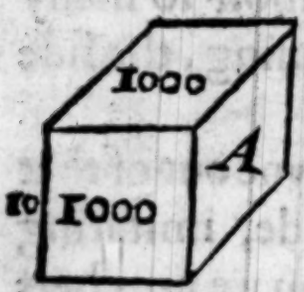
neath it one degree more towarde the left hand: and afterward diuide the nũber aboue written by the triple of the ſquare, ſetting the quotient by it ſelf, and then multiply the diuiſor by the quotient found out, and the tripled ſquare by the ſquare of the quotient, and the quotient cubically, ſubducting the products (ſo orderly added together, that ech figure may anſwere the numbers whereof it was multiplied) from the number giuen: and renew this laſt manner of diuiſion ſo many times as there are points remaining, the ſide of the Cube ſhalbe found out.

This artificiall deuiſe is drawne out of that Theoreme which Ramus made, imitating that of Euclide concerning ſquare numbers in this manner. *If a right line be cut into two ſegments, the Cube of the whole line ſhalbe equall to the Cubes of the ſegments, and to the two ſolide figures comprehended three times vnder the ſquare of his ſegment, & the ſegment remaining.*



As the line c i which is 13, is cut into two ſegments 10 & 3: the cube therefore of the whole line, namely 2197, is equall vnto the Cubes of the Segments, namely vnto 1000 and 27, and alſo to the two ſolides or parallelipipedons thriſe taken, whereof three haue a like ſoliditie, the

Soliditie of each of the three leſſer is 90, being made of the ſquare of the Segment 3, that is to ſay of 9 multiplied by the other Segment 10. Theſe three parallelipipedons iointly taken together make 270. But of the three greater parallelipipedons ech containeth 300, being made of 100, the ſquare of the greater ſegment 10 multiplied by the leſſer ſegment 3, and they beeing taken iointly together make 900.



The cube of the greater ſegment 10.

The cube of the leſſer ſegment 3,

The greater Parallelipipedons.

The leſſer parallelipipedons.

The cube therefore hath eight particular ſolides in numbers which are made of the partes of the number giuen, namely of 10 and 3 in this manner. Firſt let there be foure plane numbers made, each part being multiplied by it ſelfe, and one by another:

10	3
10	3

9
30
30
100

If againe you multiply theſe planes by the ſame partes, there will ariſe 8 ſolides, as you ſee here:

9	9	
30	30	
30	30	
100	100	
3	10	
27	90	All theſe being added
90	300	together are equall
90	300	vnto 2197, the cube of
300	1000	the whole line.

The ſame way therefore that is kept in making the Cube, is alſo to be followed in reſoluing the Cube. As for example, I marke the cube giuen with points in this manner, 2197. Then I ſubduct the particular cube of the number ſet vnder neath the laſt point: but for ſo much as that number is no Cube, I take the neareſt to it, namely an vnitie, which alſo I ſet in the quotient. This vnitie in the number giuen is 1000, but in the quotient it is but 10. The vnitie ſubducted from 2, the re-

mainder is 1, which muſt be written ouer the number giuen. So that the greater cube A is to be ſuppoſed to be ſubducted from the number giuen. This is the firſt ſteppe of this worke:

1

z 1 9 7 (1

z

I triple the quotient found out (that is to ſay, I multiply it by 3) this triple repreſenteth the three ſides taken iointly together of the three leſſer ſolides marked with C, I place the tripled number vnder 9. Againe I multiply the quotient ſquarewiſe, and triple the product which maketh likewiſe 3. This product reſembleth the three ſquared ſides taken iointly together of the three greater ſolides, marked with D, I place the product one degree lower toward the left hande vnderneath 1. With it I diuide 11, which is written aboue it, the quotient is 3. This ſegment or quotient 3, being multiplied by 3 the diuiſor, maketh 9, which in reſpect of the place wherein it ſtandeth is 900, and repreſenteth the three greater ſolides marked with D taken iointly together. Furthermore the ſame quotient being multiplyed ſquarewiſe maketh 9, and multiplied afterwarde by the tripled number ſtanding vnder 9, it maketh 27, which in reſpect of the place wherein it ſtandeth is 270, and repreſenteth the three leſſer

ſolides

ſolides marked with C. Laſt of all, the ſame quotient multiplyed cubically, breedeth the leſſer Cube B. Theſe three products therefore being added together, and the totall ſubducted from the numbers ſtanding ouer it, there remaineth nothing, which importeth that the number giuen is a Cube. The example is as you ſee:

```
x
xx97 (13
x  3
 3
---
 9
 27
 27
---
xx97
```

or otherwiſe thus:

```
 x
 x
 x
```

repeated as often as there are pointes remaining. The firſt ſteppe to finde out the ſide is in this manner.

8

1 6 3 8 7 0 6 4 (2

8

Moreouer triple the quotient found out, the product is 6, which are to be placed vnder 8, namely vnder the figure following the next pricke toward the right hande. Then multiplie the quotient by this tripled number (or which is all to one purpoſe, ſquare the quotient, and then triple the products) it maketh 12, ſet that number in a lower place one degree nearer to the left hand: and make it the diuiſor. Diuide 83 by 12, obſeruing this rule in choſing your quotient, that it be no greater, then that the numbers afterwarde produced by multiplication may not exceede the numbers ſtanding ouer it. So that here you ſhal take 1 in 8, but fiue times. Afterward by this number 5, multiply the diuiſor 12, and by the ſquare of 5 multiply the trebled number 6, and laſt of all, multiply 5 cubically, ſo ſhall you produce three numbers, namely 60, 150, 125, to be deſcribed as you ſee: Theſe numbers added togeather, and ſubducted from 8387, the remainder is 762. The ſeconde ſteppe to finde out the ſide is in this manner:

8762

8 7 6 2
1 6 3 8 7 0 6 4 (25

8 6
1 2

6 0
1 5 0
1 2 5

7 6 2 5

And becauſe there is yet one point remaining, this laſt manner of diuiſion muſt be wrought againe. Firſt therefore I triple the quotient, the product is 75, which muſt be ſo placed, that the firſt figure thereof, namely 5, may ſtand vnder 6, the ſecond vnder the 0. Againe multiply the quotient by this tripled number (or which is all one, ſquare the quotient, and triple the product) it maketh 1875, which ſhall be the diuiſor, whoſe firſt figure namely 5, muſt be placed vnder 7, the laſt figure of the tripled number. Then you ſee that 1 may be contayned in 7 many times, but I can take it but foure times, I ſet 4 in the quotient, and multiply the diuiſor by 4, the product is 7500, afterward I ſquare 4, it maketh 16, which I multiply by the tripled nūber 75, the product is 1200. Laſt of all, I multiply 4 cubically it maketh 64: theſe products added all together make 762064, this number being ſubducted frō the Cube, there remaineth nothing, whereby I gather that

the laſt figure in the quotient

the number giuen is a iuſt Cube. The third ſteppe to finde out the ſide is in this manner:

```
      7 6 2
  1 6 3 8 7 0 6 4 (254
  ---------------
            7 5
        1 8 7 5
        -------
        7 5 0 0
          1 2 0 0
                6 4
        -------------
        7 6 2 0 6 4
```

Behold alſo the example following:

```
6 1 4 1 2 5 0 0 0 (850
```

Another manner of working:

Hitherto the princely high way to finde out the ſide of the Cube hath beene declared.

But there are other waies alſo bending thereto, and leaning to the ſame principles: whereof this is one.

Hauing found out in the Table of the ſimple cubes, the figure repreſenting the ſide of the cube contained in the number ſtanding vnder the firſt point on the left hand, ſet it in the quotient, and ſubduct the particular cube of that figure as you did before: then ſquare that figure, and triple the ſquare, the product ſhall be the diuiſor, whoſe firſt figure ſhall be

be ſet vnder that figure which is on the right hand next of all to the firſt point aforeſaide.

2 See how many times the diuiſor is contained in the number written ouer it, multiply the diuiſor by the quotient, and ſubduct the product from the diuidend. Yet here you muſt take heede, that you chooſe not a greater quotient, then that the products made afterward thereby may be ſubducted from the numbers giuen.

3 When the ſubduction is done, triple the firſt figure which was ſet in the quotient, and place the laſt number which was ſet in the quotient on the right hand of the product.

4 Multiply this whole number by the ſquare of the quotient laſt found out, and ſet downe the product ſo, that the firſt figure thereof toward the right hand may ſtand vnder the point next going before on the ſame hand, and then ſubduct it from the number giuen.

As in 804357 the particular Cube, namely, 729, being taken from the number ſtanding vnder the laſt period vpon the left hand there remaine 75357, the ſide of that particular cube is 9, which I ſet in the quotient. I ſquare that ſide, it maketh 81, and triple the ſquare, the product 243 is my diuiſor, which I ſet vnder the number giuen, ſo that 3 may ſtand vnder 3. With this diuiſor diuide the

number ſtanding ouer it, you ſhall finde 2 to be contained in 7 three times. I ſet therefore 3 in the quotient, and multiply the diuiſor by it, the product is 729, which being ſubducted from 753, the remainder is 24. As you ſee in this induction.

2
7 5 4
8 0 4 3 5 7 (93
2 4 3
7 2 9

Moreouer I triple 9, the product is 27, by the which on the right hand I ſet 3 the quotient laſt found out, the totall is 273. This nũber I multiply by 9 the ſquare of 3 the quotient laſt found out, the product is 2457, which being ſubducted from the number ſtanding ouer it, there remaineth nothing. The induction is on this manner:

2 4
8 0 4 3 5 7 (93
2 7 3
9
2 4 5 7

The ſelfe ſame worke may be diſpatched another way ſomewhat differing from the former in this maner. The figure in the quotient being found out by ſubducting the particular cube, and alſo the ſecond figure in the quotient

quotient being found by diuiſion, let the whole quotient be tripled, and let the tripled number be multiplied by the former figure in the quotient. Then let the product be multiplied againe by the latter figure found out, and let a cypher be ſet on the right hande of that product. Laſt of all, let the cube of the latter figure found out be added to this product, and let the totall ſumme be ſubducted from the number giuen. As in 373248. The firſt induction is in this manner:

3 0
3 7 3 2 4 8 (7
3 4 3

Moreouer I ſquare the ſide found out, it maketh 49 and triple the ſquare, the product is 147, which ſhall be the diuiſor, by this I diuide 302, the number written ouer it, the quotient is 2. Now I triple the whole quotient 72, it maketh 216, and multiply this triple by 7, the former figure in the quotient, the product is 1512. I multiply alſo this product by 2 the latter figure of the quotient, and ſet a cypher on the right hande of it, ſo that it maketh 30240, vnto this number I adde 8 the cube of the latter figure found out, the totall is 30248, which being ſubducted from the figures aboue it, there remaineth nothing. The induction is thus:

number ſtanding ouer it, you ſhall finde 2 to be contained in 7 three times. I ſet therefore 3 in the quotient, and multiply the diuiſor by it, the product is 729, which being ſubducted from 753, the remainder is 24. As you ſee in this induction.

```
        2
      7 5 4
    8 0 4 3 5 7 (93
    ---------------
      2 4 3
      -----
      7 2 9
```

Moreouer I triple 9, the product is 27, by the which on the right hand I ſet 3 the quotient laſt found out, the totall is 273. This nũber I multiply by 9 the ſquare of 3 the quotient laſt found out, the product is 2457, which being ſubducted from the number ſtanding ouer it, there remaineth nothing. The induction is on this manner:

```
        2 4
    8 0 4 3 5 7 (93
    ---------------
          2 7 3
              9
        -------
        2 4 5 7
```

The ſelfe ſame worke may be diſpatched another way ſomewhat differing from the former in this maner. The figure in the quotient being found out by ſubducting the particular cube, and alſo the ſecond figure in the quotient

quotient being found by diuiſion, let the whole quotient be tripled, and let the tripled number be multiplied by the former figure in the quotient. Then let the product be multiplied againe by the latter figure found out, and let a cypher be ſet on the right hande of that product. Laſt of all, let the cube of the latter figure found out be added to this product, and let the totall ſumme be ſubducted from the number giuen. As in 373248. The firſt induction is in this manner:

3 0
3 7 3 2 4 8 (7
3 4 3

Moreouer I ſquare the ſide found out, it maketh 49 and triple the ſquare, the product is 147, which ſhall be the diuiſor, by this I diuide 302, the number written ouer it, the quotient is 2. Now I triple the whole quotient 72, it maketh 216, and multiply this triple by 7, the former figure in the quotient, the product is 1512. I multiply alſo this product by 2 the latter figure of the quotient, and ſet a cypher on the right hande of it, ſo that it maketh 30240, vnto this number I adde 8 the cube of the latter figure found out, the totall is 30248, which being ſubducted from the figures aboue it, there remaineth nothing. The induction is thus:

$$\begin{array}{r} 3\ 0 \\ \cancel{3}\ \cancel{7}\ \cancel{3}\ 2\ 4\ 8\ (27 \\ \hline 1\ 4\ 7 \\ \hline 3\ 0\ 2\ 4\ 8 \end{array}$$

All the points of the number giuen being examined, if any thing remaine, it ſignifieth that the number giuen is no Cube: wherefore the true ſide of it cannot be giuen exactly in numbers. Yet if it pleaſe you to ſift out the neareſt ſide that may be, by the firſt kinde of reduction of mixt numbers, you muſt reduce the number giuen into a cubicall fraction of a great denomination, and afterwarde ſeeke out the cubicall ſide of that fraction.

For example ſake, becauſe 120 is no Cube, therefore let it be reduced into ſixtie cubicall parts, after this manner. Multiply 60 cubically in it ſelfe, it maketh 216000, by this being taken for the denominator of the fraction, multiply 120 the number giuen, the product is 25920000, whoſe cubicall ſide is $\frac{295}{60}$ that is, $4\frac{11}{12}$ the neareſt to the true ſide that may be.

FINIS.